Answer on Question #73473, Physics / Mechanics | Relativity

Question. The equation of motion of a damped harmonic oscillator is given by $\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0$. $m = 0.25\ kg$, $b = 0.14\ \frac{1}{s}$ and $\omega_0 = 18.4\ \frac{1}{s}$ Calculate

i) the time period;

ii) number of oscillations in which its amplitude will become half of its initial value;

iii) number of oscillations in which its mechanical energy will reduce to half of its initial value.

Solution.

i) If $\omega_0 = 18.4\ \frac{1}{s}$ then

$$\omega = 2\pi\nu = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}} = \frac{2 \cdot 3.14}{\sqrt{18.4^2 - 0.14^2}} = 0.341\ s.$$

ii) The amplitude of a damped harmonic oscillator is

$$A = A_0 e^{-bt}.$$

So

$$\frac{A_0}{2} = A_0 e^{-0.14t} \rightarrow \frac{1}{2} = e^{-0.14t} \rightarrow t = -\frac{\ln\frac{1}{2}}{0.14} = 4.951\ s;$$

$$N_1 = \frac{t}{T} = \frac{4.951}{0.341} \approx 14.$$

iii) Mechanical energy of oscillations

$$E = \frac{m\omega^2 A^2}{2}.$$

We have

$$E_0 = \frac{m\omega^2 A_0^2}{2};$$

$$\frac{E_0}{2} = \frac{m\omega^2 A^2}{2};$$

$$\frac{E_0}{\frac{E_0}{2}} = \frac{m\omega^2 A_0^2}{2} \cdot \frac{2}{m\omega^2 A^2} \quad \rightarrow \quad 2 = \frac{A_0^2}{A^2} \quad \rightarrow \quad A = \frac{A_0}{\sqrt{2}}.$$

Hence (see ii))

$$t = -\frac{\ln\frac{1}{\sqrt{2}}}{0.14} \approx 2.47\ s;$$

$$N_2 = \frac{t}{T} = \frac{2.47}{0.341} \approx 7.$$

Answer. $T = 0.341\ s$; $N_1 = 14$; $N_2 = 7$.