Answer on Question 73308, Physics, Mechanics, Relativity

Question:

A child of mass 50 kg is standing on the edge of a merry-go-round of mass 250 kg and radius 3.0 m which is rotating with an angular velocity of 3.0 rad/s^{-1}. The child then starts walking towards the centre of the merry-go-round. What will be the final angular velocity of the merry-go-round when the child reaches the centre?

Solution:

We can find the final angular velocity of the merry-go-round from the law of conservation of angular momentum:

$$L_i = L_f,$$

$$I_i \omega_i = I_f \omega_f,$$

here, I_i is the initial rotational inertia of the system, I_f is the final rotational inertia of the system, ω_i is the initial angular velocity of the merry-go-round, $\omega_f$is the final angular velocity of the merry-go-round.

We can find the initial rotational inertia of the system as follows:

$$I_i = \left(I_{disk,i} + I_{child,i}\right) = \left(\frac{1}{2} m_{disk} r_{disk,i}^2 + m_{child} r_{child,i}^2\right),$$

here, $I_{disk,i} = \frac{1}{2} m_{disk} r_{disk,i}^2$ is the initial rotational inertia of the merry-go-round, $I_{child,i} = m_{child} r_{child,i}^2$ is the initial rotational inertia of the child, m_{disk} is the mass of the marry-go-round, m_{child} is the mass of the child, r_{disk} is the radius of the merry-go-round, r_{child} is the distance from the centre of the merry-go-round to the child.

Then, we can calculate I_i:

$$I_i = \left(\frac{1}{2} m_{disk} r_{disk,i}^2 + m_{child} r_{child,i}^2\right) =$$

$$= \left(\frac{1}{2} \cdot 250\ kg \cdot (3.0\ m)^2 + 50\ kg \cdot (3.0\ m)^2\right) = 1575\ kg \cdot m^2.$$

Similarly, we can find the final rotational inertia of the system:

$$I_f = \left(I_{disk,f} + I_{child,f}\right) = \left(\frac{1}{2} m_{disk} r_{disk,f}^2 + m_{child} r_{child,f}^2\right),$$

here, $I_{disk,f} = \frac{1}{2} m_{disk} r_{disk,f}^2$ is the final rotational inertia of the merry-go-round, $I_{child,f} = m_{child} r_{child,f}^2$ is the final rotational inertia of the child, m_{disk} is the mass of the marry-go-round, m_{child} is the mass of the child, r_{disk} is the radius of the merry-go-round, r_{child} is the distance from the centre of the merry-go-round to the child.

Then, we can calculate I_f:

$$I_f = \left(\frac{1}{2} m_{disk} r_{disk,f}^2 + m_{child} r_{child,f}^2\right) =$$

$$= \left(\frac{1}{2} \cdot 250\ kg \cdot (3.0\ m)^2 + 50\ kg \cdot (0.0\ m)^2\right) = 1125\ kg \cdot m^2.$$

Finally, we can calculate the final angular velocity of the merry-go-round from the law of conservation of angular momentum:

$$I_i \omega_i = I_f \omega_f,$$

$$\omega_f = \omega_i \frac{I_i}{I_f} = 3.0\ \frac{rad}{s} \cdot \frac{1575\ kg \cdot m^2}{1125\ kg \cdot m^2} = 4.2\ \frac{rad}{s}.$$

Answer:

$\omega_f = 4.2\ \frac{rad}{s}$.