Answer on Question 73280, Physics / Mechanics | Relativity

Question

A wheel 2.0 m in diameter lies in the vertical plane and rotates about its central axis with a constant angular acceleration of 4.0 rad s−2 . The wheel starts at rest at t = 0 and the radius vector of a point A on the wheel makes an angle of 60º with the horizontal at this instant. Calculate the angular speed of the wheel, the angular position of the point A and the total acceleration at t = 2.0s.

Solution. The equation that gives the angular speed $\omega(t)$ of the wheel is

$$\omega(t) = \omega_0 + \alpha t$$

where ω_0 is the initial angular speed, $\omega_0 = 0$, α is the angular acceleration, $\alpha = 4.0\, rad/s^2$. Substituting knowing numbers, we find the angular speed of the wheel at t = 2.0s.

$$\omega(t = 2\text{ s}) = 0 + 4.0\ \frac{rad}{s^2} \cdot 2.0\, s = 8.0\ \ \frac{rad}{s}$$

The equation that gives the angular position of the point A is

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

where $\theta_0 = 60°$ is the initial angular position of the radius vector of a point A. Substituting knowing numbers, we find the angular position of the point A at t = 2.0 s

$$\theta(t = 2) = 60° + 0 + \frac{1}{2} \cdot 4.0\ \frac{rad}{s^2} \cdot (2.0\, s)^2 = 60° + 8.0\, rad$$

Define how many degrees are equal to 8 radians. It is known that $1\, rad = 57.2958°$, then

$$8\, rad = 8 \cdot 57.2958° \approx 458.0°$$

and

$$\theta(t = 2\, s) \approx 60° + 458.0° = 518.0°$$

In order to find the total acceleration, we first find the tangential a_τ and normal a_n acceleration at t = 2.0s

$$a_\tau(t = 2\, s) = \alpha r = 4.0\ \frac{rad}{s^2} \cdot 1.0\, m = 4.0\ \frac{m}{s^2}$$

$$a_n(t = 2\, s) = r\omega^2 = 1.0\, m \cdot \left(8.0\ \ \frac{rad}{s}\right)^2 = 64.0\ \frac{m}{s^2}$$

The equation that gives the total acceleration is

$$a_{tot} = \sqrt{a_\tau^2 + a_n^2}$$

Substituting knowing numbers, we find

$$a_{tot}(t = 2\, s) = \sqrt{(4.0)^2 + (64.0)^2} = \sqrt{16 + 4096} \approx 64.125\ \frac{m}{s^2}$$

Answer:

$$\omega(t = 2\text{ s}) = 8.0\ \ \frac{rad}{s}$$

$$\theta(t = 2\, s) \approx 518.0°$$

$$a_{tot}(t = 2\, s) \approx 64.125\ \frac{m}{s^2}$$