Answer on Question #73153-Physics-Other

Prove that the volume of a unit cell of the reciprocal lattice of a bcc structure is inversely proportional to the volume of the unit cell of its direct lattice.

Solution

For a bcc structure:

$$\boldsymbol{a_1} = \frac{a}{2}(-1,1,1)$$

$$\boldsymbol{a_2} = \frac{a}{2}(1,-1,1)$$

$$\boldsymbol{a_3} = \frac{a}{2}(1,1,-1)$$

$$V_b = \boldsymbol{a_1} \times \boldsymbol{a_2} \cdot \boldsymbol{a_3}$$

$$\boldsymbol{a_1} \times \boldsymbol{a_2} = \frac{a}{2}(-1,1,1) \times \frac{a}{2}(1,-1,1) = \frac{a^2}{4}(2,2,0)$$

$$V_b = \frac{a^2}{4}(2,2,0) \cdot \frac{a}{2}(1,1,-1) = \frac{a^3}{8}(2+2) = \frac{a^3}{2}.$$

For the reciprocal lattice:

$$\boldsymbol{a_1^*} = \frac{c}{a}(0,1,1)$$

$$\boldsymbol{a_2^*} = \frac{c}{a}(1,0,1)$$

$$\boldsymbol{a_3^*} = \frac{c}{a}(1,1,0)$$

$$V^* = \boldsymbol{a_1^*} \times \boldsymbol{a_2^*} \cdot \boldsymbol{a_3^*}$$

$$\boldsymbol{a_1^*} \times \boldsymbol{a_2^*} = \frac{c}{a}(0,1,1) \times \frac{c}{a}(1,0,1) = \frac{c^2}{a^2}(1,1,-1)$$

$$V^* = \frac{c^2}{a^2}(1,1,-1) \cdot \frac{c}{a}(1,1,0) = \frac{1}{2}\frac{c^3}{a^3} = \frac{1}{4}\frac{c^3}{V_B}$$

Thus,

$$V^* \sim \frac{1}{V_B}$$