Answer on Question #73115, Physics / Mechanics | Relativity

Question. What is the effect of damping in an oscillatory system? Differentiate between heavy and critical damping. Show that the displacement of a weakly damped oscillator is given by $x(t) = A \cdot exp(-\beta t) \cdot cos(\omega t - \varphi)$ where symbols have their usual meanings.

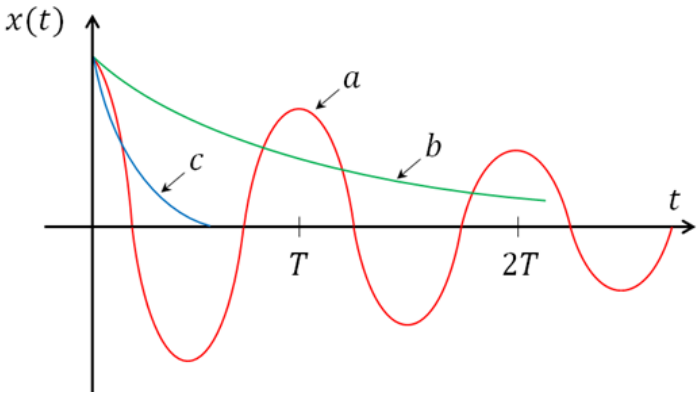
Solution.

1. What is the effect of damping in an oscillatory system?

As a result of the damping, the oscillatory system loses energy and the amplitude is decreased.

2. Differentiate between heavy and critical damping.

Damping can be ***light***, in which case the system oscillates about the midpoint (a), ***heavy***, in which the system takes a long time to reach equilibrium (b) or ***critical***, where the system reaches equilibrium in a short time compared with T with no overshoot, where T is the natural period of vibration of the system (c).



3. Show that the displacement of a weakly damped oscillator is given by $x(t) = A \cdot exp(-\beta t) \cdot cos(\omega t - \varphi)$ where symbols have their usual meanings.

The unforced damped harmonic oscillator has equation

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0.$$

It has characteristic equation

$$\lambda^2 + 2\beta\lambda + \omega_0^2\lambda = 0$$

with characteristics roots

$$\lambda_{1,2} = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega_0^2}}{2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}.$$

There are three cases depending on the sign of the expression under the square root:

i) $\beta^2 < \omega_0^2$ (this will be ***light*** damping, β is small relative to ω_0).

ii) $\beta^2 > \omega_0^2$ (this will be ***heavy*** damping, β is large relative to ω_0).

iii) $\beta^2 = \omega_0^2$ (this will be ***critical*** damping, β is just between ***heavy*** and ***light*** damping).

Case (i) ***Light*** damping (non-real complex roots)

If $\beta^2 < \omega_0^2$ then the term under the square root is negative and the characteristic roots are not real. In order for $\beta^2 < \omega_0^2$ the constant β must be relatively small. Let $\omega = \sqrt{\omega_0^2 - \beta^2}$. Then we have characteristic roots

$$-\beta \pm i\omega$$

leading to complex exponential solutions:

$$e^{(-\beta+i\omega)t}, e^{(-\beta-i\omega)t}.$$

The basic real solution are

$$e^{-\beta t} \cos \omega t \quad and \quad e^{-\beta t} \sin \omega t.$$

The general real solution is found by taking linear combinations of the two basic solutions, that is

$$x(t) = C_1 e^{-\beta t} \cos \omega t + C_2 e^{-\beta t} \sin \omega t$$

or

$$x(t) = e^{-\beta t}(C_1 \cos \omega t + C_2 \sin \omega t) = A e^{-\beta t} \cos(\omega t - \phi).$$

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