Answer on Question #73029 Physics / Mechanics | Relativity

The amplitude of an oscillator is $x_{\max} = 8$ cm and it completes $N = 100$ oscillations in $\tau = 80$ s. i) Calculate its time period T and angular frequency ω. ii) If the initial phase is $\varphi_0 = \pi/4$, write expressions for its displacement $x(t)$ and velocity $v(t)$. iii) Calculate the values of maximum velocity $v_{\max}$ and acceleration $a_{\max}$.

Solution:

The period of oscillation

$$T = \frac{\tau}{N} = \frac{80}{100} = 0.8 \text{ s}$$

The angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 2.5\pi \text{ rad/s}$$

The oscillator displacement

$$x(t) = x_{\max}\cos(\omega t + \varphi_0) = 8\cos\left(2.5\pi t + \frac{\pi}{4}\right) \text{ cm}$$

The velocity

$$v(t) = x'(t) = 20\pi \sin\left(2.5\pi t + \frac{\pi}{4}\right)\frac{\text{cm}}{\text{s}}$$

The acceleration

$$a(t) = v'(t) = 50\pi^2 \cos\left(2.5\pi t + \frac{\pi}{4}\right) \text{ cm/s}^2$$

The maximum velocity

$$v_{\max} = 20\pi\,\frac{\text{cm}}{\text{s}} = 62.8\,\frac{\text{cm}}{\text{s}}$$

The maximum acceleration

$$a_{\max} = 50\pi^2\,\frac{\text{cm}}{\text{s}^2} = 493\,\frac{\text{cm}}{\text{s}^2}$$

Answers:

i) 0.8 s, 2.5π rad/s

ii) $8\cos\left(2.5\pi t + \frac{\pi}{4}\right)$ cm, $20\pi \sin\left(2.5\pi t + \frac{\pi}{4}\right)\frac{\text{cm}}{\text{s}}$

iii) $62.8\ \frac{\text{cm}}{\text{s}}$, $493\ \frac{\text{cm}}{\text{s}^2}$