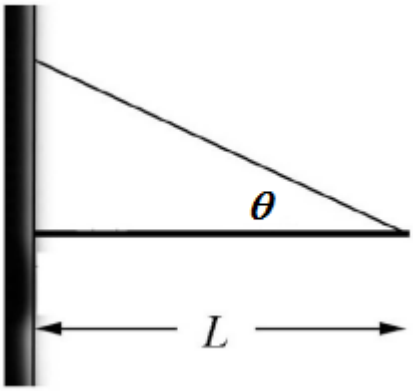
Answer on Question #72610, Physics / Mechanics | Relativity

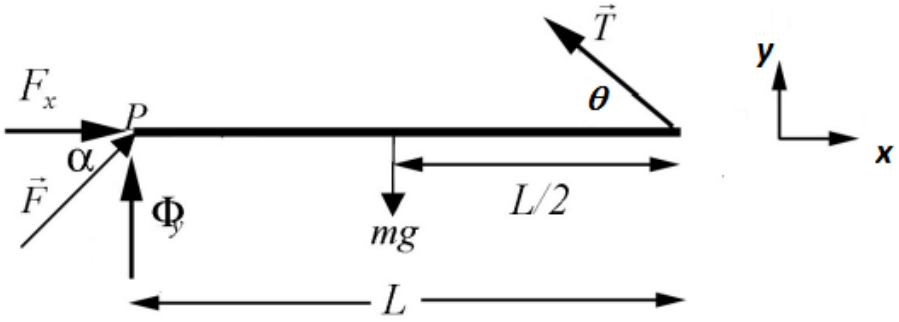
The horizontal rod with a mass of 10 kg and length 12 m is hinged to a wall at one end and supported by a cable which makes an angle of 30º with the rod at its other end.

Calculate the tension in the cable and the force exerted by the hinge.

Solution:



The force diagram is shown in the figure below



For static equilibrium, the sum of the forces must be zero, and hence the sums of the components of the forces must be zero;

$$-Tcos\theta + Fcos\alpha = 0$$

$$-mg + Tsin\theta + Fsin\alpha = 0$$

Next we find the torques with respect to the hinge point. With respect to the hinge point, and taking positive torques to be counterclockwise, the gravitational force exerts a negative torque of magnitude $mgL/2$ and the cable exerts a positive torque of magnitude $TLsin\theta$. Setting the net torque equal to zero then gives

$$TLsin\theta - mg\frac{L}{2} = 0$$

So,

$$T = \frac{mg}{2sin\theta} = \frac{(10\ kg)(9.8\ m/s^2)}{2\sin 30^\circ} = 98\ N$$

Substituting the above expression for the tension into the force equations yields

$$F\cos\alpha = T\cos\theta = \frac{mg}{2\tan\theta}$$

$$F\sin\alpha = mg - T\sin\theta = \frac{mg}{2}$$

From the above, dividing one equation by the other, we see that

$$\tan\alpha = \tan\theta$$

The horizontal forces on the rod must cancel. The tension force and the pivot force act with the same angle (but in opposite horizontal directions) and hence must have the same magnitude

$$F = T = 98\ N$$

Answer: $F = T = 98\ N$.