Answer on Question # 71942, Physics / Mechanics | Relativity

Question

a) A gas is contained in a vessel of volume 0.02 m3 at a pressure of 300 kPa and a temperature of 15°C. The gas is passed into a vessel of volume 0.015 m3. Determine to what temperature the gas must be cooled for the pressure to remain the same

b) The piston of an air compressor compresses air to 1⁄4 of its original volume during its stroke.

Determine the final pressure of the air if the original pressure is 100 kPa, assuming an isothermal change. The characteristic gas constant for air is 287 Jkg−1K−1

Solution. To solve these problems we use the ideal gas law

$$PV = nRT \qquad (1)$$

where P is the absolute pressure of a gas, V is the volume it occupies, n is the number of moles, R is the universal gas constant, T is the absolute temperature (in units of Kelvins) of a gas

a) We know the initial volume $V_0 = 0.02\ \text{m}^3$, the initial pressure $P_0 = 300\text{kPa} = 3 \cdot 10^5$ Pa, the initial temperature $T_0 = 15°C$, the final volume $V_f = 0.015\ \text{m}^3$ and the final pressure which is the same as the initial pressure $P_f = P_0\ \ = 3 \cdot 10^5$ Pa. We must find the final temperature T_f. Use the equation (1) twice

$$P_0 V_0 = nRT_0$$

$$P_f V_f = nRT_f$$

Divide $P_0 V_0$ by $P_f V_f$

$$\frac{P_0 V_0}{P_f V_f} = \frac{n_0 R T_0}{n_f R T_f}$$

Since the pressure is constant, P_f and P_0 are the same and they cancel out. The same is true for n_f and n_0, as well as for R, which is a constant. Therefore

$$\frac{V_0}{V_f} = \frac{T_0}{T_f}$$

and we have Charles' law. Find T_f

$$T_f = T_0 \frac{V_f}{V_0}$$

Convert temperature T_0 from Celsius to Kelvin.

$$T_0 = (15.0 + 273)\text{K} = 288\ \text{K}$$

Substitute the known values into the equation

$$T_f = 288\ K \cdot \frac{0.015\ \text{m}^3}{0.02\ \text{m}^3} = 216\ K$$

Convert temperature T_f from Kelvin to Celsius

$$T_f = (216 - 273)\ °C = -57\ °C$$

b) We know the initial pressure $P_0 = 100\text{kPa}$, the final volume $V_f = (1/4)V_0$ and final temperature is equal to initial temperature, $T_f = T_0$. We must find the final pressure P_f. Use the equation (1) twice

$$P_0 V_0 = nRT_0$$
$$P_f V_f = nRT_f$$

Divide $P_0 V_0$ by $P_f V_f$

$$\frac{P_0 V_0}{P_f V_f} = \frac{n_0 R T_0}{n_f R T_f}$$

Since the temperature is constant T_f and T_0 cancel out. The same is true for n_f and n_0, as well as for R, which is a constant. Therefore

$$\frac{P_0 V_0}{P_f V_f} = 1$$

or

$$P_0 V_0 = P_f V_f$$

and we have Boyle's law. Find P_f

$$P_f = P_0 \frac{V_0}{V_f}$$

Substitute the known values into the equation

$$P_f = 100\text{ kPa } \frac{V_0}{(1/4)V_0} = 400\text{ kPa}$$

Answer:

a) The gas must be cooled for $T_f = -57\ °C$

b) The final pressure of the air is $P_f = 400\text{ kPa}$