Answer to Question #71543, Physics / Mechanics | Relativity

In a stamping machine the die has a mass of $30\ kg$ and falls freely a distance of $2\ m$ to strike a metal block. The depth of indentation is $10mm$. Assuming that the die does not rebound determine the average resistance of the metal. You are required to compare and contrast by solving this problem in two ways:
a) by making use of the principle of conservation of momentum and D'Alembert's principle
b) by making use of the principle of conservation of energy.

Solution.

We have:

$$m = 30\ kg - mass\ of\ the\ die$$

$$h = 2m - distance\ of\ die\ falling$$

$$s = 0.01\ m - depth\ of\ indentation$$

a) Find speed of the die at the moment of touching with block:

$$h = \frac{gt^2}{2}$$

$$t = \sqrt{\frac{2h}{g}}$$

$$v = gt = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 2} = 6.26\ m/s$$

By D'Alembert's principle:

$$ma = mg - F_{res}$$

where F_{res} is the average resistance of the metal.

Then:

$$a = \frac{dv}{dt}\ ; v = \frac{ds}{dt} \Rightarrow a = \frac{dv}{ds}v$$

$$mvdv = (mg - F_{res})ds$$

$$\frac{mv^2}{2} = (mg - F_{res})s$$

Answer:

$$F_{res} = \left| mg - \frac{mv^2}{2s} \right| = \left| 30 \cdot 9.8 - \frac{30 \cdot 6.26^2}{2 \cdot 0.01} \right| = 58487\ N$$

b) By the principle of conservation of energy:

$$mg(h + s) = F_{res}s$$

Answer:

$$F_{res} = \frac{mg(h + s)}{s} = \frac{30 \cdot 9.8 \cdot (2 + 0.01)}{0.01} = 59094\ N$$