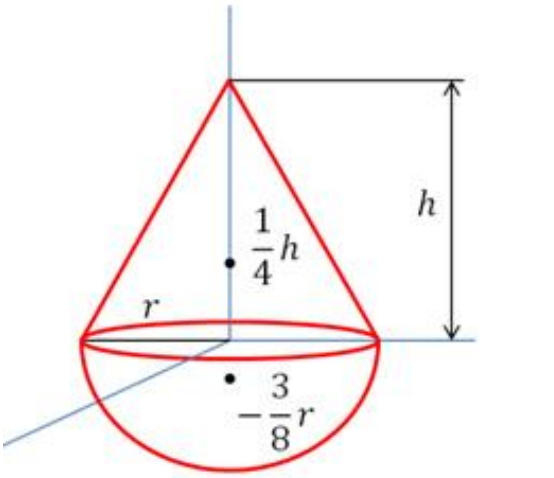
Answer on Question #71153, Physics / Mechanics | Relativity

Question. A uniform toy of constant density is made by mounting a cone of height h and radius r on a hemisphere of radius r. Find ratio h/r if center of mass of toy is to toy is to lie at the center of common base.

Solution.



According to the formula

$$Z_C = \frac{m_1 \cdot Z_{C_1} + m_2 \cdot Z_{C_2}}{m_1 + m_2},$$

where m_1 – the mass of a cone; m_2 – the mass of a hemisphere; Z_{C_1} – the center of mass of a cone; Z_{C_2} – the center of mass of a hemisphere. Hence

$$Z_C = \frac{m_1 \cdot Z_{C_1} + m_2 \cdot Z_{C_2}}{m_1 + m_2} = \frac{\frac{1}{3}\pi r^2 h\rho \cdot \frac{1}{4}h - \frac{2}{3}\pi r^3 \rho \cdot \frac{3}{8}r}{\frac{1}{3}\pi r^2 h\rho + \frac{2}{3}\pi r^3 \rho} = \frac{\frac{1}{3}h \cdot \frac{1}{4}h - \frac{1}{4}r^2}{\frac{1}{3}h + \frac{2}{3}r}$$

Since $Z_C = 0$ then

$$0 = \frac{\frac{1}{3}h \cdot \frac{1}{4}h - \frac{1}{4}r^2}{\frac{1}{3}h + \frac{2}{3}r} \quad \rightarrow \quad \frac{1}{3}h \cdot \frac{1}{4}h - \frac{1}{4}r^2 = 0 \quad \rightarrow \quad \frac{h}{r} = \sqrt{3}$$

Answer. $\frac{h}{r} = \sqrt{3}$.