Answer on Question #70942, Physics / Molecular Physics | Thermodynamics

Question. A fluid at $0.7\,bar$ occupying $0.09\,m^3$ is compressed reversibly to a pressure of $3.5\,bar$ according to law $pV^n = constant$. The fluid is then heated reversibly at constant volume until the pressure is $4\,bar$; the specific volume is then $0.05\,m^3/kg$. A reversible expansion according to a law $pV^2 = constant$ restores the fluid to its initial state. Sketch the cycle to a $p-V$ diagram and calculate:

i) The mass of fluid present (THIS is the one I cannot seem to figure out.. basics be damned).

ii) the value of n in the first process.

iii) the net work of the cycle.

Given.

$p_1 = 0.7\,bar = 70000\,Pa$, $V_1 = 0.09\,m^3$, $pV^n = constant$;

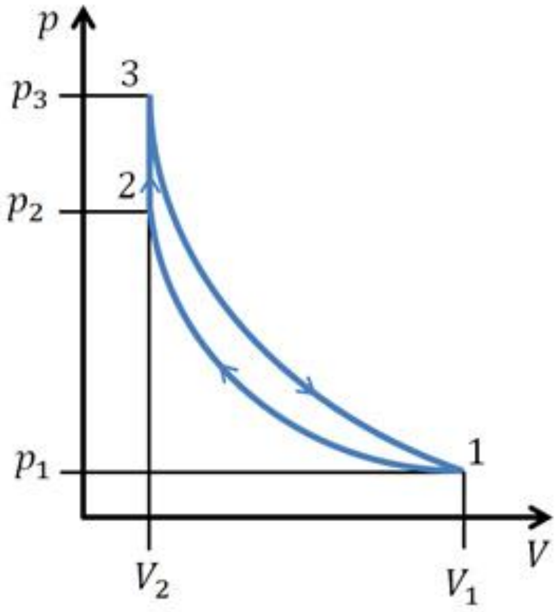
$p_2 = 3.5\,bar = 350000\,Pa$;

$p_3 = 4\,bar = 400000\,Pa$;

$V_2 = V_3$;

$pV^2 = constant$;

$v = 0.05\,m^3/kg$.



Find.

$m, n, A-?$.

Solution.

i) For $1-2$

$$p_1V_1^n = p_2V_2^n. \qquad (1)$$

Since $V_2 = V_3$ then for $3-1$ we have

$$p_3V_3^2 = p_3V_2^2 = p_1V_1^2 \rightarrow V_2 = V_3 = V_1\sqrt{\frac{p_1}{p_3}} = 0.09\sqrt{\frac{0.7}{4}} = 0.0376\,m^3.$$

So

$$m = \frac{V_3}{v} = \frac{0.0376}{0.05} = 0.753\,kg.$$

ii) From (1)

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^n \to \ln\left(\frac{p_1}{p_2}\right) = \ln\left(\frac{V_2}{V_1}\right)^n \to \ln\left(\frac{p_1}{p_2}\right) = n\ln\frac{V_2}{V_1} \to n = \frac{\ln\left(\frac{p_1}{p_2}\right)}{\ln\frac{V_2}{V_1}} = \frac{\ln\frac{0.7}{3.5}}{\ln\frac{0.0376}{0.09}} = 1.84.$$

iii) The net work of the cycle

$$A = A_{1-2} + A_{2-3} + A_{3-1}$$

$$A_{1-2} = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{p_1 V_1^{1.84}}{V^{1.84}} dV = p_1 V_1^{1.84} \int_{0.09}^{0.0376} \frac{1}{V^{1.84}} dV = 70000 \cdot 0.09^{1.84} \cdot \frac{V^{-1.84+1}}{-1.84+1}\bigg|_{0.09}^{0.0376} =$$

$$= \frac{833.5}{-0.84} \cdot \left(\frac{1}{V^{0.84}}\bigg|_{0.09}^{0.0376}\right) = \frac{833.5}{-0.84} \cdot \left(\frac{1}{0.0376^{0.84}} - \frac{1}{0.09^{0.84}}\right) \approx -8108\, J.$$

$$A_{2-3} = \int_{V_2}^{V_3} p dV = 0.$$

$$A_{3-1} = \int_{V_3}^{V_1} p dV = \int_{V_3}^{V_1} \frac{p_1 V_1^2}{V^2} dV = p_1 V_1^2 \int_{0.0376}^{0.09} \frac{1}{V^2} dV = 70000 \cdot 0.09^2 \cdot \frac{V^{-2+1}}{-2+1}\bigg|_{0.0376}^{0.09} =$$

$$= \frac{567}{-1} \cdot \left(\frac{1}{V}\bigg|_{0.0376}^{0.09}\right) = \frac{567}{-1} \cdot \left(\frac{1}{0.09} - \frac{1}{0.0376}\right) \approx 8780\, J.$$

Finally

$$A = -8108 + 0 + 8780 = 672\, J.$$

Answer. $m = 0.753\, kg; n = 1.84; A = 672\, J.$