Answer on Question #70652, Physics / Mechanics | Relativity

Question. If the crown is not pure gold, then how much silver is in it?

Assume that the volume of the crown is $64.8\ cm^3$ and that its mass is $1000\ g$. We also know that $1000\ g$ of gold has a volume of about $51.8\ cm^3$ and $1000\ g$ of silver has a volume of about $95.2\ cm^3$. If the crown is not pure gold that how much silver is in it?

Given.

$m = 1000\ g; V = 64.8\ cm^3; \rho_1 = \frac{1000}{51.8} = 19.3\ g/cm^3; \rho_2 = \frac{1000}{95.2} = 10.5\ g/cm^3.$

Find.

$m_2 - ?.$

Solution.

The mass of the crown is

$$m = m_1 + m_2,$$

where m_1 – the mass of gold in the crown and m_2 – the mass of silver in the crown. Then

$$m = \rho_1 V_1 + \rho_2 V_2,$$

where V_1 is the volume of gold in the crown and V_2 is the volume of silver in the crown.

So

$$\frac{m}{V} = \rho_1 \frac{V_1}{V} + \rho_2 \frac{V_2}{V} = \rho_1 \frac{V - V_2}{V} + \rho_2 \frac{V_2}{V} = \rho_1 \left(1 - \frac{V_2}{V}\right) + \rho_2 \frac{V_2}{V};$$

$$\frac{m}{V} = \rho_1 - \rho_1 \frac{V_2}{V} + \rho_2 \frac{V_2}{V} \rightarrow \frac{m}{V} = \rho_1 - \frac{V_2}{V}(\rho_1 - \rho_2) \rightarrow V_2 = \frac{\rho_1 - \frac{m}{V}}{\rho_1 - \rho_2} V \rightarrow \rho_2 V_2 = \frac{\rho_1 - \frac{m}{V}}{\rho_1 - \rho_2} V \rho_2 \rightarrow$$

$$\rightarrow \boldsymbol{m_2 = \frac{\rho_1 - \frac{m}{V}}{\rho_1 - \rho_2} V \rho_2}.$$

Finally

$$m_2 = \frac{\rho_1 - \frac{m}{V}}{\rho_1 - \rho_2} V \rho_2 = \frac{19.3 - \frac{1000}{64.8}}{19.3 - 10.5} \cdot 64.8 \cdot 10.5 \approx 300\ g.$$

Answer. $\boldsymbol{m_2 = \frac{\rho_1 - \frac{m}{V}}{\rho_1 - \rho_2} V \rho_2}$. For our assumption $m_2 \approx 300\ g$.