Answer on Question #70566, Physics / Electromagnetism

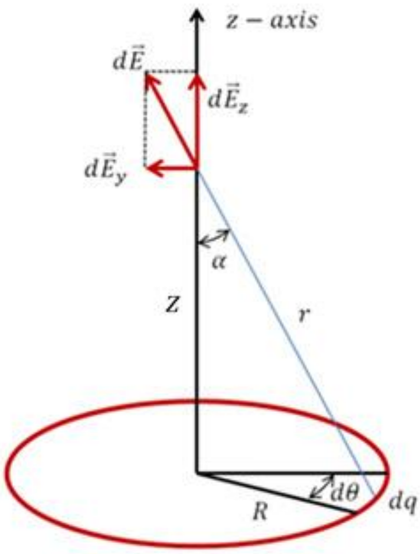
Question. An electron is constrained to the central axis of a ring of charge 1 coulomb and of radius 1 meter. The electrostatic force exerted on the electron can cause the electron to oscillate through the center of the ring. Find the frequency of small oscillation of the electron in Hertz.

Given. $Q = 1\,C, R = 1\,m, |e| = 1.6 \cdot 10^{-19}\,C, m = 9.1 \cdot 10^{-31}\,kg.$

Find. ν—?

Solution.

First of all, let's find the electric field at some distance z along the z-axis as shown in fig. It is necessary to notice that all of the horizontal components of the electric field will cancel out, leaving only the vertical components. Thus, we can use the cosine function to get just the vertical component of the field.



So

$$dE_z = dE\cos\alpha = \frac{dq}{4\pi\varepsilon_0 r^2}\cos\alpha,$$

$$dq = ds \cdot \lambda = R \cdot d\theta \cdot \lambda,$$

where $\lambda = \frac{Q}{2\pi R}$ is the charge density of the ring and $ds = R \cdot d\theta$ is arc length. Then

$$dq = R \cdot d\theta \cdot \frac{Q}{2\pi R} = \frac{Qd\theta}{2\pi}.$$

The distance r we can calculate using the Pythagorean theorem

$$r^2 = R^2 + z^2.$$

The cosine function in terms of z and R can be written as

$$\cos\alpha = \frac{z}{r} = \frac{z}{\sqrt{R^2 + z^2}}.$$

Putting everything together gives us an expression for dE_z

$$dE_z = \frac{Qz}{4\pi\varepsilon_0 \cdot 2\pi} \frac{d\theta}{(R^2 + z^2)^{3/2}}.$$

Finally

$$E_z = \int_0^{2\pi} \frac{Qz}{4\pi\varepsilon_0 \cdot 2\pi} \frac{d\theta}{(R^2 + z^2)^{3/2}} = \frac{Qz}{4\pi\varepsilon_0 \cdot 2\pi} \frac{2\pi}{(R^2 + z^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0} \frac{Qz}{(R^2 + z^2)^{3/2}}.$$

Suppose we are only interested in the electric field very close to the centre of the ring. In other words, $R \gg z$. Then, the R^2 term in the denominator would completely dwarf the z^2 term in the denominator. Thus, we can simplify the expression further for small distances (small z) by dropping z in the denominator.

$$E_z \approx \frac{1}{4\pi\varepsilon_0} \frac{Qz}{(R^2 + 0)^{\frac{3}{2}}} = \frac{Qz}{4\pi\varepsilon_0 R^3}.$$

An electron that is released very close to the centre of a positively charged ring (along the z-axis) will feel a restoring force. Thus, it will oscillate about the centre of the ring. According to Newton's second law

$$\vec{F} = m\vec{a}.$$

Then

$$-\frac{Qze}{4\pi\varepsilon_0 R^3} = ma \quad or \quad -\frac{Qze}{4\pi\varepsilon_0 R^3} = m\frac{d^2z}{dt^2},$$

$$m\frac{d^2z}{dt^2} + \frac{Qez}{4\pi\varepsilon_0 R^3} = 0,$$

$$\frac{d^2z}{dt^2} + \frac{Qe}{4\pi\varepsilon_0 m R^3} z = 0.$$

This is simply the equation for oscillatory motion. We can let $\omega^2 = \frac{Qe}{4\pi\varepsilon_0 m R^3}$. We get

$$\frac{d^2z}{dt^2} + \omega^2 z = 0.$$

So

$$\omega = \sqrt{\frac{Qe}{4\pi\varepsilon_0 m R^3}}.$$

$$\omega = 2\pi\nu \quad \rightarrow \quad \nu = \frac{\omega}{2\pi}.$$

Finally

$$\nu = \frac{1}{2\pi}\sqrt{\frac{Qe}{4\pi\varepsilon_0 m R^3}} = \frac{1}{2\pi}\sqrt{\frac{1 \cdot 1.6 \cdot 10^{-19}}{4\pi \cdot 8.85 \cdot 10^{-12} \cdot 9.1 \cdot 10^{-31} \cdot 1^3}} \approx 6.3 \cdot 10^9\ Hz.$$

Answer: $\nu = \frac{1}{2\pi}\sqrt{\frac{Qe}{4\pi\varepsilon_0 m R^3}} \approx 6.3 \cdot 10^9\ Hz.$

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