Answer on Question #70531, Physics / Electromagnetism

Question. Derive the resonance condition for nuclear magnetic resonance. Calculate the require magnetic field strength at which proton spin comes into resonance with $500\ MHz$ radiation.

Given.

$\nu_L = 500\ \text{MHz}.$

Find.

$B_0 - ?.$

Solution.

In a few words (*in more detail see D. Freude Spectroscopy*), the energy of a magnetic moment $\vec{p}_m$ in a magnetic field $\vec{B}_0$ is given by:

$$E = -\vec{p}_m \cdot \vec{B}_0.$$

If the z axis is chosen along $\vec{B}_0$ then

$$E = -p_{mz} B_0$$

or

$$E = -\gamma m \frac{h}{2\pi} B_0,$$

where γ is the gyromagnetic ratio, m is the magnetic quantum number of the atomic nucleus, h is the Planck constant. For uneven mass numbers $m = \pm\frac{1}{2}$ and we get two levels with an energy difference of

$$\Delta E = \gamma \frac{h}{2\pi} B_0.$$

The energy absorbed by the proton is then $E = h\nu_L$, where ν_L is the resonance radio frequency. Hence, a magnetic resonance absorption will only occur when $\Delta E = h\nu_L$, which is when

$$\nu_L = \frac{\gamma}{2\pi} B_0.$$

This most important equation of NMR relates the magnetic fields to the resonant frequency.

For the proton $\frac{\gamma}{2\pi} = 42.58\ MHz/T$.

Finally

$$B_0 = {\nu_L}\Big/{\frac{\gamma}{2\pi}} = \frac{500}{42.58} = 11.74\ T.$$

Answer: $B_0 = 11.74\ T$.