Answer to Question #70494, Physics / Mechanics | Relativity

A length S is divided into n equal parts, at the end of which, the acceleration of a moving point is increased by f/n. Find the velocity of the point after describing the distance s given that the particle started from rest with accelerating f. EXPLAIN

Solution: first of all we should think how the speed changes from the start of the part till the end of the part. We can simply write

$$v_i = v_{i-1} + t_i(f + \frac{f}{n}i)$$

Where v_{i-1} is the speed of the point when it enters this i-th part, ti is the time that is needed to cover the part, and $f + \frac{f}{n}i$ is the acceleration on the i-th part (f – is the initial acceleration, $\frac{f}{n}i$ – the increase gathered up to i-th part inclusively)

Now

$$t_i = \frac{\frac{s}{n}}{\frac{v_{i-1} + v_i}{2}}$$

Because the speed changes linearly when the point moves over the i-th part.

We should also note that I changes from 0 to n-1

Now we combine both:

$$t_i = \frac{\frac{s}{n}}{\frac{v_{i-1} + v_{i-1} + t_i(f + \frac{f}{n}i)}{2}} = \frac{2\frac{s}{n}}{2v_{i-1} + t_i(f + \frac{f}{n}i)}$$

$$2v_{i-1}t_i + t_i^2\left(f + \frac{f}{n}i\right) - 2\frac{s}{n} = 0$$

We solve this for t:

$$t_i = \frac{-2v_{i-1} \pm \sqrt{4v_{i-1}^2 + 8\frac{s}{n}\left(f + \frac{f}{n}i\right)}}{2\left(f + \frac{f}{n}i\right)}$$

Now t can not be negative, so only the positive root is used

$$t_i = \frac{\sqrt{4v_{i-1}^2 + 8\frac{s}{n}\left(f + \frac{f}{n}i\right)} - 2v_{i-1}}{2\left(f + \frac{f}{n}i\right)}$$

On the other hand we know that

$$v_i = v_{i-1} + t_i(f + \frac{f}{n} i)$$

So

$$v_i = v_{i-1} + \frac{\sqrt{4v_{i-1}^2 + 8\frac{s}{n}\left(f + \frac{f}{n} i\right)} - 2v_{i-1}}{2\left(f + \frac{f}{n} i\right)} \left(f + \frac{f}{n} i\right) =$$

$$v_i = \frac{\left(\sqrt{4v_{i-1}^2 + 8\frac{s}{n}\left(f + \frac{f}{n} i\right)}\right)}{2}$$

Now we need to get rid of v_{i-1}^2. It is obvious that $4v_{i-1}^2 = 4v_{i-2}^2 + 8\frac{s}{n}\left(f + \frac{f}{n}(i-1)\right)$, and so on until

$$i - k = 0$$

So we can write

$$v_i = \frac{\left(\sqrt{8\frac{s}{n} f + 8\frac{s}{n}\left(f + \frac{f}{n}\right) + 8\frac{s}{n}\left(f + \frac{f}{n} * 2\right) + \cdots + 8\frac{s}{n}\left(f + \frac{f}{n}(i-1)\right) + 8\frac{s}{n}\left(f + \frac{f}{n} i\right)}\right)}{2}$$

$$v_i = \sqrt{2s} \sqrt{\left(f + \left(f + \frac{f}{n}\right) + \left(f + \frac{f}{n} * 2\right) + \cdots + \left(f + \frac{f}{n}(i-1)\right) + \left(f + \frac{f}{n} i\right)\right)\frac{1}{n}}$$

Now if you need you can calculate any v_i depending on your n but I think you wish to know what $v_i$when $n \to \infty$.

$$\sqrt{\left(f + \left(f + \frac{f}{n}\right) + \left(f + \frac{f}{n} * 2\right) + \cdots + \left(f + \frac{f}{n}(i-1)\right) + \left(f + \frac{f}{n} i\right)\right)\frac{1}{n}}$$

$$= \sqrt{\left(f + \left(f + \frac{f}{n}\right) + \cdots + \left(f + \frac{f}{n}(n-1)\right)\right)\frac{1}{n}}$$

$$= \sqrt{f} \sqrt{\left(1 + \left(1 + \frac{1}{n}\right) + \cdots + \left(1 + \frac{1}{n}(n-1)\right)\right)\frac{1}{n}}$$

There are n sums, with one in every sum so

$$\sqrt{f}\sqrt{\left(1+\left(1+\frac{1}{n}\right)+\cdots+\left(1+\frac{1}{n}(n-1)\right)\right)\frac{1}{n}}=\sqrt{f}\sqrt{1+\sum_{1}^{n-1}\frac{i}{n^2}}$$

$\sum_{10}^{n-1}\frac{i}{n^2}$ is a simple algebraic sum (n is constant, don't mind it, only I matters), so

$$\sum_{1}^{n-1}\frac{i}{n^2}=(1+n-1)*\frac{n-1}{2n^2}=\frac{(n^2-n)}{2n^2}=\frac{1}{2}-\frac{1}{2n^2}$$

But if n is infinity then

$$\frac{1}{2n^2}=0$$

So

$$\boldsymbol{v_\infty=\sqrt{2s}\sqrt{f}\sqrt{1+\sum_{1}^{n-1}\frac{i}{n^2}}=\sqrt{2sf}\left(\sqrt{1+\frac{1}{2}}\right)=\sqrt{3sf}}$$

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