Answer on Question #70388, Physics - Mechanics | Relativity

What volume of iron of density 7.8 g/cm per cubic must be attached to wood whose mass is 100 g if wood has a density of 0.3 g/cm per cubic and both have to be submerged in water?

Solution:

Because the both objects is at equilibrium under the water, the net force on it must be zero, therefore the upward force must balance the downward force. You can write this using Newton's 2nd Law in the y-direction as:

$$F_{net} = F_B - mg = 0$$

The buoyancy force is

$$F_B = \rho_{water} V g$$

density of water = 1.00 g/cm^3

The volume is

$$V = V_1 + V_2$$

$$V_1 = \frac{m_{wood}}{\rho_{wood}} = \frac{100\ g}{0.3\ g/cm^3} = 333.3\ cm^3$$

$$V_2 = \frac{m_{iron}}{\rho_{iron}}$$

Hence,

$$\rho_{water}\left(\frac{m_{wood}}{\rho_{wood}} + \frac{m_{iron}}{\rho_{iron}}\right) g = (m_{wood} + m_{iron}) g$$

$$m_{iron}\left(1 - \frac{\rho_{water}}{\rho_{iron}}\right) = m_{wood}\left(\frac{\rho_{water}}{\rho_{wood}} - 1\right)$$

$$m_{iron}\left(1 - \frac{1}{7.8}\right) = m_{wood}\left(\frac{1}{0.3} - 1\right)$$

$$m_{iron} = m_{wood}\frac{\left(\frac{1}{0.3} - 1\right)}{\left(1 - \frac{1}{7.8}\right)} = 2.68 m_{wood} = 2.676 \times 100 = 267.6\ g$$

The volume of iron is

$$V_{iron} = \frac{m_{iron}}{\rho_{iron}} = \frac{267.6}{7.8} = 34.3\ cm^3$$

Answer: $34.3\ cm^3$