Answer on Question #70291, Physics / Mechanics | Relativity

A particle with initial velocity v=-2i+4j in meters per second at t=0 undergoes a constant acceleration (vector a) of magnitude a= 3meter per square second at an angle of 130degrees from the positive direction of the x axis. What is the particle's velocity (vector v) at t=5s, in unit vector notation and as a magnitude and an angle?

SOLUTION

Uniform acceleration is a type of motion in which the velocity of an object changes by an equal amount in every equal time period. There is a simple formula relating the time-dependent velocity to the time elapsed:

$\vec{v}(t) = \overrightarrow{v_0} + \vec{a}t$, (1)

where t is the elapsed time,

$\overrightarrow{v_0}$ is the initial velocity, and

$\vec{a}$ is the uniform rate of acceleration.

The unit vectors in the direction of the x and y axes of a two dimensional Cartesian coordinate system are i and j. In our case $\overrightarrow{v_0} = -2\text{i} + 4\text{j}$. Let us assume that the values are written in SI. The vector of acceleration has a magnitude of $|\vec{a}| = 3$, but the vector is turned at an angle of 130° from the positive direction of the x axis. Thus its projections to the x and y axes are $a_x = |\vec{a}| \cos 130° = 3 \cos 130°$, $a_y = |\vec{a}| \sin 130° = 3 \sin 130°$, besides, velocity's projections are $v_x = -2, v_y = 4$ (see Fig. 1).

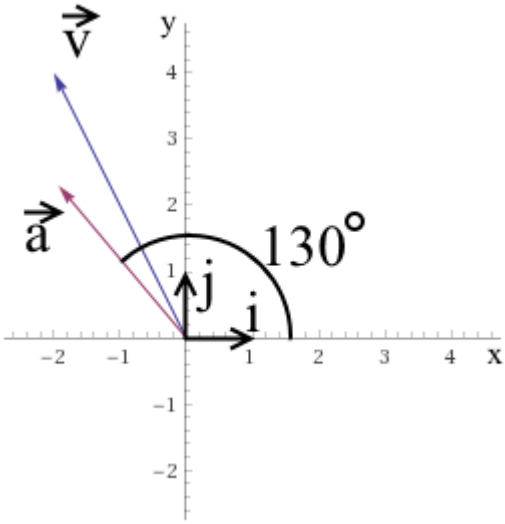

Fig. 1. Vectors of acceleration (a) and initial velocity (v)

Equation (1) can be rewritten in a projection representation:

$$\begin{matrix} v_x(t) = v_{0x} + a_x t \\ v_y(t) = v_{0y} + a_y t \end{matrix} \quad (2)$$

All necessary values are known, we have to find the projections of velocity at t=5:

$$v_x(t) = -2 + 3 \cos 130° \cdot 5 \approx -11.642$$
$$v_y(t) = 4 + 3 \sin 130° \cdot 5 \approx 15.491$$

Therefore, $\vec{v} = v_x i + v_y j = -11.642i + 15.491j$ (in unit vectors notation) (see Fig. 2). Its magnitude and angle from the positive direction of x axis can be determined as:

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{11.642^2 + 15.491^2} = 19.378$$

$$\alpha = \text{atan}\frac{v_y}{v_x} = \text{atan}\frac{15.491}{-11.642} = 126.926°$$

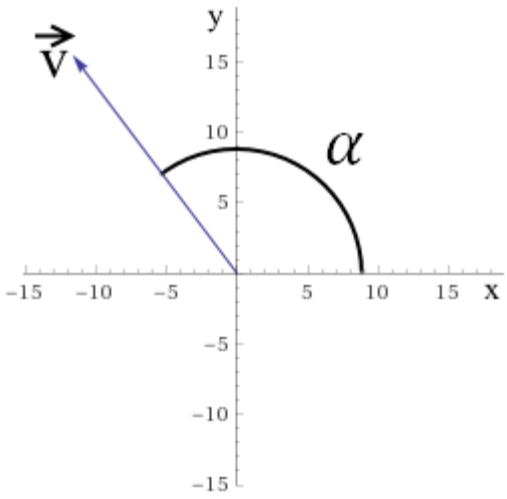

Fig. 2. Velocity's vector at t=5 s

ANSWER: the particle's velocity (vector v) at t=5 s is $\vec{v} = -11.642i + 15.491j$ in meters per second in unit vector notation. A magnitude is 19.378 m/s and it is directed at the angle of 126.926° from the positive direction of the x axis.