Answer on Question # 70191, Physics / Electromagnetism

Question. A non-conducting spherical shell, with an inner radius of R_1 and an outer radius of R_2, has charge spread non-uniformly through its volume between its inner and outer surfaces, where the volume charge density ρ is given by $\rho = k/r$ with the distance r from the center of the shell and a constant k.

(a) Find the net charge Q of the shell;

(b) Calculate the electric field E at a distance R from the center over the range $R_1 < R < R_2$;

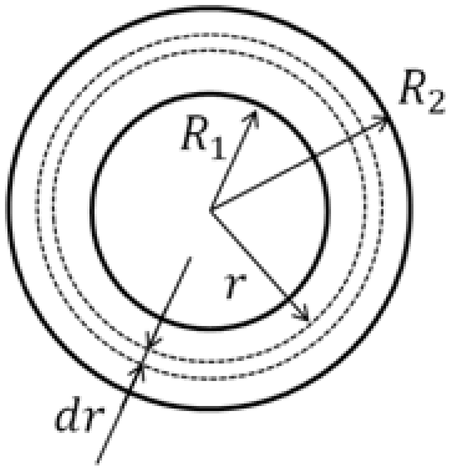
(c) Find the maximum E_{max}.

Given.

R_1;

R_2;

$\rho = k/r$.



Find.

Q;

E_r;

E_{max}.

Solution.

a) Let us consider a sphere of radius $r > R_1$ (see figure) then charge, enclosed by the considered sphere,

$$Q = \int_{R_1}^{R_2} 4\pi r^2 \cdot \rho \cdot dr = \int_{R_1}^{R_2} 4\pi r^2 \cdot \frac{k}{r} \cdot dr = \int_{R_1}^{R_2} 4\pi r \cdot k \cdot dr = 4\pi k \cdot \frac{r^2}{2}\Big|_{R_1}^{R_2} = 2\pi k\left({R_2}^2 - {R_1}^2\right).$$

b) Now, applying Gauss' theorem,

$$E_r \cdot 4\pi r^2 = \frac{Q_{enclosed}}{\varepsilon_0} \ (where\ E_r\ is\ the\ projection\ of\ electric\ field\ along\ the\ radial\ line).$$

For $0 < r \leq R_1$:

$$Q_{enclosed} = 0 \quad \rightarrow \quad E_r = 0.$$

For $R_1 < r \leq R_2$:

$$Q_{enclosed} = \int_{R_1}^{r} 4\pi r^2 \cdot \frac{k}{r} \cdot dr = \int_{R_1}^{r} 4\pi r \cdot k \cdot dr = 4\pi k \cdot \frac{r^2}{2}\bigg|_{R_1}^{r} = 2\pi k\left(r^2 - {R_1}^2\right);$$

$$E_r = \frac{2\pi k\left(r^2 - {R_1}^2\right)}{4\pi r^2 \varepsilon_0} = \frac{k\left(r^2 - {R_1}^2\right)}{2r^2\varepsilon_0} = \frac{k}{2\varepsilon_0}\left(1 - \frac{{R_1}^2}{r^2}\right)$$

For $R_2 < r$:

$$Q_{enclosed} = 2\pi k\left({R_2}^2 - {R_1}^2\right);$$

$$E_r = \frac{2\pi k\left({R_2}^2 - {R_1}^2\right)}{4\pi r^2 \varepsilon_0} = \frac{k\left({R_2}^2 - {R_1}^2\right)}{2r^2\varepsilon_0} = \frac{k\left({R_2}^2 - {R_1}^2\right)}{2\varepsilon_0} \cdot \frac{1}{r^2}.$$

c) As magnitude of electric field decreases with increasing r for $R_2 < r$, field will be maximum for $R_1 < r \le R_2$. Now, for E_r to be maximum

$$E_{max} = \frac{k}{2\varepsilon_0}\left(1 - \frac{{R_1}^2}{{R_2}^2}\right).$$

Answer.

a) $Q = 2\pi k\left({R_2}^2 - {R_1}^2\right)$.

b) for $0 < r \le R_1$ $E_r = 0$;

for $R_1 < r \le R_2$ $E_r = \frac{k}{2\varepsilon_0}\left(1 - \frac{{R_1}^2}{r^2}\right)$;

for $R_2 < r$ $E_r = \frac{k\left({R_2}^2 - {R_1}^2\right)}{2\varepsilon_0} \cdot \frac{1}{r^2}$.

c) $E_{max} = \frac{k}{2\varepsilon_0}\left(1 - \frac{{R_1}^2}{{R_2}^2}\right)$.