Answer on Question #70087, Physics / Other

Question A chain hangs over a nail with 2.0 m on one side and 6.0 m on the other side. If the force of friction is equal to the weight of 1.0 m of the chain, calculate the time required for the chain to slide off the nail.

Solution Let us use following notation. The forces that acts on the rope are:

$$F_g = \rho g L, \quad F_l = \rho g l, \quad F_f = \rho g \cdot f$$

where F_g is force from longer part ($L = 6$), F_l is force from shorter ($l = 2$) and F_f is force from friction, proportional to $f = 1$ m of chain. Let us denote total length $L_0 = L + l$.

Now we can write down balance of forces:

$$F_{tot} = F_g - F_l - F_f = ma = \rho L_0 a$$

$$\rho g L - \rho g l - \rho g \cdot f = \rho L_0 a$$

$$g(L - l - f) = L_0 a$$

We also remember that resulting acceleration describes the change of L:

$$a = \frac{d^2 L}{dt^2}$$

So now we have differential equation for $L(t)$:

$$\frac{d^2 L}{dt^2} = g\frac{L - l - f}{L_0} = g\frac{L - (L_0 - L) - f)}{L_0} = g\frac{2L - (L_0 + f)}{L_0}$$

The general solution of this equation is

$$L(t) = C_1 + C_2 e^{\sqrt{\frac{2g}{L_0}}}$$

Using initial condition $L(0) = 6$ and $dL/dt(0) = 0$ we obtain solution:

$$L = \frac{L_0 + f}{2} + \frac{L + 0/2 - f}{2} e^{\sqrt{\frac{2g}{L_0}}}$$

From this we can find time of falling is ($L = 8$):

$$t = \sqrt{\frac{8}{2 \cdot 9.8}} \ln(7/3) \approx 0.54\, s$$