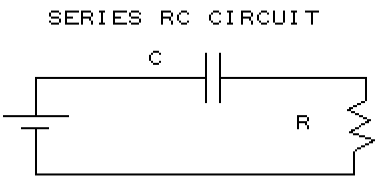
Answer on Question #70084, Physics / Other

An emf of 100 V is applied to a series RC circuit in which the resistance is 200 ohms and the capacitance is 10^{-4} farads. Determine the charge q(t) on the capacitor if q(0) = 0. Also determine the current i(t).

Solution:



Applying the loop law to this circuit,

$$EMF - iR - \frac{q}{C} = 0$$

where is the terminal voltage of the battery, i (lower case because it varies with time) times R is the voltage drop across the resistor, and q (lower case because it varies with time) divided by C is the potential cross the capacitor. The battery source voltage is the EMF.

Rearranging:

$$EMF = iR + \frac{q}{C}$$

We cannot solve this equation as it stands because both i and q are unknowns. However, they are related through calculus by the equation:

$$i = \frac{dq}{dt}$$

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{emf}{R}$$

Substituting

$$\frac{dq}{dt} + \frac{10^4 q}{200} = \frac{100}{200}$$

This is a linear differential equation of first-order. The solution of is

$$q = emfC(1 - e^{(-t/RC)})$$

$$q = 100\times10^{-4}(1 - e^{(-\frac{t}{200\times10^{-4}})})$$

$$q(t) = 0.1(1 - e^{(-50t)})$$

Differentiating the above equation would yield us current.

$$i(t) = \frac{emf}{R}e^{(-t/RC)} = 0.5e^{(-50t)}$$

Answer: $q(t) = 0.1(1 - e^{(-50t)})$; $i(t) = 0.5e^{(-50t)}$