Answer on Question #70065-Physics-Other

A random variable X has the following probability distribution: f(x) = kx(1-x) for 0<x<1 and f(x) = 0, elsewhere Determine the value of the constant k and E(X).

Solution

$$\int_0^1 \mathrm{kx(1-x)dx} = 1$$

$$k = \frac{1}{\int_0^1 \mathrm{x(1-x)dx}}$$

$$\int_0^1 \mathrm{x(1-x)dx} = \int_0^1 (\mathrm{x - x^2})\mathrm{dx} = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

Thus,

$$k = \frac{1}{\frac{1}{6}} = 6.$$

$$E(X) = 6\int_0^1 \mathrm{x(1-x)dx}\,(x) = 6\int_0^1 \mathrm{x^2(1-x)dx}$$

$$\int_0^1 \mathrm{x^2(1-x)dx} = \int_0^1 (\mathrm{x^2 - x^3})\mathrm{dx} = \left(\frac{x^3}{3} - \frac{x^4}{4}\right)_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}.$$

Thus,

$$E(X) = \frac{6}{12} = \frac{1}{2}.$$