Answer on Question #70062-Physics-Other

Using Green's Theorem evaluate the integral ∫subscript C {(Y^3) dx - (x^3) dy} where C is a circle of radius 3 units centered at the origin.

Solution

Using Green's Theorem

$$I = \oint \{(\mathrm{Y}^3)\,\mathrm{dx} - (\mathrm{x}^3)\,\mathrm{dy}\} = \iint_D \left(\frac{\partial(-(\mathrm{x}^3))}{\partial x} - \frac{\partial((\mathrm{y}^3))}{\partial y}\right) dA$$

$$\left(\frac{\partial(-(\mathrm{x}^3))}{\partial x} - \frac{\partial((\mathrm{y}^3))}{\partial y}\right) = -3(x^2 + y^2) = -3r^2.$$

In polar coordinates:

$$dA = rdrd\theta$$

$$I = -3\int_0^{2\pi} d\theta \int_0^3 r^2 r dr.$$

$$\int_0^{2\pi} d\theta = 2\pi - 0 = 2\pi$$

$$\int_0^3 r^3 dr = \left(\frac{r^4}{4}\right)_0^3 = 3^4 - 0^4 = 3^4.$$

Thus,

$$I = -\frac{3(2\pi)3^4}{4} = -\frac{3^5\pi}{2} = -\frac{243\pi}{2}.$$

Answer: $-\frac{243\pi}{2}$.

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