Answer on Question #70060-Physics-Other

Using the divergence theorem evaluate ∫∫subscript s F^vector. dS^vector where F^vector = (y^2)z×i^cap + (y^3) j^cap+ xzk^cap and S is the surface of the cube defined by - 1< = x<=1 :-1<=y<=1 : 0<=z<=2.

Solution

Using the divergence theorem:

$$I = \oiint \boldsymbol{F} d\boldsymbol{S} = \iiint div\, \boldsymbol{F}\, dV$$

$$div\, \boldsymbol{F} = \frac{\partial}{\partial x}((y^2)\, z) + \frac{\partial}{\partial y}(y^3) + \frac{\partial}{\partial z}(xz) = 0 + 3y^2 + x = x + 3y^2$$

$$I = \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2} dz\, (x + 3y^2) = \int_{-1}^{1} xdx \int_{-1}^{1} dy \int_{0}^{2} dz + \int_{-1}^{1} dx \int_{-1}^{1} 3y^2 dy \int_{0}^{2} dz$$

$$= \int_{-1}^{1} xdx\, \big(1 - (-1)\big)(2 - 0) + \big(1 - (-1)\big) \int_{-1}^{1} 3y^2 dy\, (2 - 0) = 4\left(\frac{x^2}{2}\right)_{-1}^{1} + 4(y^3)_{-1}^{1}$$

$$= 4\left(\frac{1}{2} - \frac{1}{2}\right) + 4\big(1 - (-1)\big) = 0 + 8 = 8.$$

Answer: 8.