Answer on Question #70059-Physics-Mechanics-Relativity

Calculate the work done by a force F^vector = (x-y)i^cap +xy j^cap in moving a particle counter-clockwise along the circle x^2 +y^2 =4 from the point (2, 0) to the point (0, -2).

Solution

$$\boldsymbol{r}(t) = (2\cos t\,, 2\sin t), 0 \le\ t \le \frac{3\pi}{2}.$$

$$\boldsymbol{F}(r(t)) = (2(\cos t - \sin t), 4\cos t \sin t)$$

$$W = \oint \boldsymbol{F}(r(t))\ \boldsymbol{r}'(t)dt = \int_0^{\frac{3\pi}{2}} (2(\cos t - \sin t), 4\cos t\sin t)2(-\sin t\,, \cos t)dt$$

$$W = 4\int_0^{\frac{3\pi}{2}} [-\cos t\sin t + \sin^2 t + 2\cos^2 t\sin t]dt$$

$$W = 4\left(\frac{t}{2} - \frac{1}{4}\sin 2t - \frac{\cos t}{2} + \frac{\cos^2 t}{2} - \frac{1}{6}\cos 3t\right)_0^{\frac{3\pi}{2}}$$
$$= 2\left(\left(\frac{3\pi}{2} - \frac{1}{2}\sin 3\pi - \cos\frac{3\pi}{2} + \cos^2\frac{3\pi}{2} - \frac{1}{3}\cos\frac{9\pi}{2}\right) - \left(0 - \frac{1}{2}\sin 0 - \cos 0 + \cos^2 0 - \frac{1}{3}\cos 0\right)\right)$$

$$W = 2\left(\left(\frac{3\pi}{2} - 0 - 0 + 0 - 0\right) - \left(0 - 0 - 1 + 1 - \frac{1}{3}\right)\right) = \left(3\pi + \frac{2}{3}\right).$$

Answer: $\left(3\pi + \frac{2}{3}\right)$.