# Answer on Question # 69946 - Physics - Mechanics | Relativity

## Question

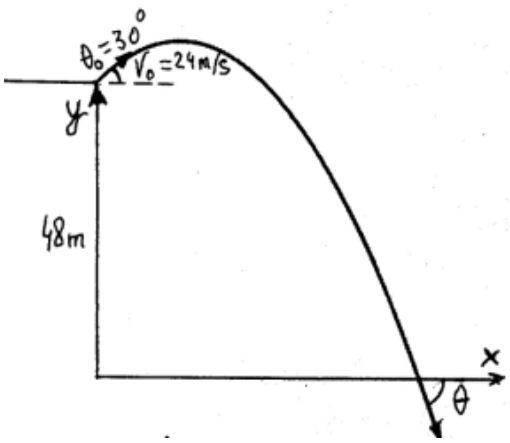
A Spear is thrown upward from a cliff 48m above the ground. Given an initial speed of 24 m/s at an angle of 30 (degrees) to the horizontal"

a. how long is the spear in flight?

b. what is the magnitude and direction of the spear's velocity just before it hits the ground?

## Solution

Resolve the motion of the spear into horizontal and vertical components along the x- and y-axes. These axes are perpendicular, so components of the initial velocity $v$ are $v_{0x} = v_0\, cos\theta_0$ and $v_{0y} = v_0 sin\theta_0$ , where $v_0 = 24\ m/s$ is the magnitude of the initial velocity and $\theta_0 = 30^0$ is its direction, as shown in Figure.




Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical. The kinematic equations for horizontal and vertical motion of the spear take the following forms:

Horizontal motion of the spear ($a_x = 0$)

$v_x = v_{0x} = 24 \cdot \cos 30^0 = 24 \cdot \frac{\sqrt{3}}{2} = 12\sqrt{3}\ m/s$ velocity is a constant.

Vertical motion of the spear (assuming positive is up $a_y = -g = -9.80 m/s^2$ due to gravitation)

$\mathrm{v_y} = \mathrm{v_{0y}} - \mathrm{gt}$

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

where $y_0 = 48\ m$is the cliff height, $v_{0y} = 24 \cdot \sin 30^0 = 12\ m/s$

First find how long is the spear in flight. When the spear falls to the ground, its y-coordinate will be 0, i.e.

$$\boldsymbol{y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 0}$$

Substitute $y_0 = 48$ , $v_{0y} = 12$

$y = 48 + 12 \cdot t - 4.9 \cdot t^2 = 0$

or

$4.9t^2 - 12t - 48 = 0$

Solve this equation

$t = \frac{12 \pm \sqrt{12^2 - 4 \cdot 4.9 \cdot (-48)}}{2 \cdot 4.9} = \frac{12 \pm \sqrt{144 + 940.8}}{9.8} \approx \frac{12 \pm 32.94}{9.8}$

$t_1 = 4.59\ s$

$t_2 = -2.14\ s$

Only the positive value of time should be considered so the spear was in flight $t = 4.59\ s$

Now we find $\mathrm{v_y}$ just before the spear hits to the ground. Substituting into the formula $\mathrm{v_y} = \mathrm{v_{0y}} - \mathrm{gt}$ values $v_{0y} = 12$ , $g = 9.8$ and $t = 4.59$ we get

$\mathrm{v_y} = 12 - 9.8 \cdot 4.59 \approx -33\ m/s$

where the minus sign indicates that the velocity is directed downward.

Find the magnitude of the spear's velocity just before it hits the ground

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(12\sqrt{3}\right)^2 + (-33)^2} = \sqrt{1521} = 39\ m/s$

Find the $\cos\theta$ where $\theta$ is an angle of the spear's velocity to the horizontal just before the spear fell to the ground

$\sin\theta = \frac{v_y}{v} = \frac{-33}{39} = \frac{-11}{13} = -0.85$

Then we get $\theta = -57.8^0$. The negative angle means that the velocity is $\theta = -57.8^0$ below the horizontal. This result is consistent with the fact that the final vertical velocity is negative.

**Answer:** a. The spear was in flight $t = 4.59\ s$.

b. The magnitude and the angle of the spear's velocity to the horizontal just before it hits the ground are $v = 39\ m/s$ and $\theta = -57.8^0$.