Answer on Question #69888-Physics-Other

A cannon that is 1m off the ground shoots a projectile 60m/s towards a target that is also 1m off the ground and is 1m tall, the cannon and the target are 100m away. At what angles do you need to shoot the projectile to hit the target?

Solution

$$x = v_x t = v\cos\alpha\, t.$$

$$t = \frac{x}{v\cos\alpha}$$

The vertical positions of the cannon and the target are the same, so we can use it as base level ($y = 0$).

The minimal distance is

$$y_{min} = 0 = v\sin\alpha\, t - \frac{gt^2}{2}$$

$$\sin\alpha = \frac{gt}{2v} = \frac{g}{2v}\frac{x}{v\cos\alpha}$$

$$\sin 2\alpha = \frac{gx}{v^2}$$

$$\alpha = \frac{1}{2}\sin^{-1}\left(\frac{gx}{v^2}\right) = \frac{1}{2}\sin^{-1}\left(\frac{9.8\cdot 100}{60^2}\right) = 32°$$

The maximal distance is

$$y_{max} = 1 = v\sin\alpha\, t - \frac{gt^2}{2}$$

$$1 = v\sin\alpha\left(\frac{x}{v\cos\alpha}\right) - \frac{g}{2}\left(\frac{x}{v\cos\alpha}\right)^2$$

$$1 = 100\tan\alpha - \frac{9.8}{2}\left(\frac{100}{60}\right)^2(1+\tan^2\alpha)$$

$$\tan\alpha_1 = 0.149 \to \alpha_1 = \tan^{-1}0.149 = 8.5°$$

$$\tan\alpha_2 = 7.198 \to \alpha_2 = \tan^{-1}7.198 = 82°.$$

The first result is unphysical, because it is less than minimal angle to reach the bottom of the target!

We need to shoot the projectile to hit the target at the angles from $32°$ to $82°$.