

Answer on Question #69702, Physics / Astronomy | Astrophysics

Ques. Derive the expression for the mean temperature in a star: $\langle T \rangle = \frac{2}{3} \frac{E_g}{E_i}$

Answer:

Internal energy $E_i = - E_g / 2$

E_g is Gravitational Energy.

$$dP/dm = - G m / (4\pi r^4)$$

Where P is the pressure, m is the mass enclosed in the spherical surface of radius r.

Gravitational Energy of the star

$$E_g = - G M^2 / R$$

Where, M = mass of the star and R = radius of the star

Let,

$$\rho = M / [4\pi R^3 / 3]$$

$$R = [3 M / (4\pi\rho)]^{1/3}$$

Internal Energy

$$E_i = 1/2 * G M^2 * [3 M / (4\pi\rho)]^{-1/3} = 1/2 * G M^{5/3} \rho^{1/3} * (3/4\pi)^{1/3}$$

Internal Energy (of a mono-atomic ideal gas or gas in the form of ions)

$$E_i = 3/2 k T N = 3/2 k T [M / \mu m_H]$$

Where, k = Boltzmann's constant, T = average temperature of the star, N = number of molecules/particles of gas, M = mass of the gas, m_H = mass of the particle of gas basically N is proportional to the mass M of star

We get:

$$E_i = M^{5/3} * \rho^{1/3} = T * M$$

$$T = M^{2/3} * \rho^{1/3}$$