Answer on Question #69478, Physics / Mechanics | Relativity

For a damped harmonic oscillator, the equation of motion is m(d2x/dt2)+y(dx/dt)+kx=0 with m=0.50 kg, y=0.70kgs-1 and k=70Nm-1. Calculate, the period of motion, number of oscillations in which it's amplitude will become half of its initial value, the number of oscillations in which it's mechanical energy will drop to half of its initial value, it's relaxation time, quality factor.

Solution:

m(d2x/dt2)+y(dx/dt)+kx=0
Where m=0.5 kg, g=0.7 kg×s-1 , k=70 N×m-1
d 2 x /dt 2 + g /m dx/ dt + k/ m x = 0
Solution of equation: x t = Ae $^{-\beta t}$ cosωt

Cyclic frequency of free oscillations: $\omega_0{}^2 = k/m$
Damped coefficient: $\beta = g/2m$
Where cyclic frequency of free damped oscillations $\omega = (\omega_0{}^2 - \beta)^{1/2}$

The period of motion:
$T = 2\pi/\omega_0$
$\omega_0 = k/m$
$T = 2\pi\,(m/k)^{1/2}$
T=0.53 s

$x_0/x_t = 2$
$e^{\beta t} = 2$ (11)
$t = (1/\beta)\ln 2$
$\beta = g/2m$
$t = (2m/g)\ln 2$
t=0.99 s

Number of oscillations in which its amplitude will become half of its initial value:
N = t /T
N=1.87

$x_0/x_t = 2$
$e^{\beta t} = 1.41$
$t = (1/\beta)\ln 1.41$
$\beta = g/2m$
$t = (2m/g)\ln 1.41$
t=0.49 s

Number of oscillations in which its mechanical energy will drop to half of its initial value:
N = t /T
N=0.93

Relaxation time:

$\tau = 1/\beta$

$\beta = g/2m$

$\tau = 2m/q$

$\tau = 1.43$ s

<u>Quality factor:</u>

$Q = \pi/\beta T$

$\beta = g/2m$

$Q = 2\pi m/gT$

$Q=8.46$

Answer: $T=0.53$ s; $N=1.87$; $N=0.93$; $\tau = 1.43$ s; $Q=8.46$