Answer on Question #69401-Physics / Other

A body of mass $m = 0.2$ kg is suspended from a spring of force constant $k = 100$ N/m. The frictional force acting on it is $F_{\text{friction}} = 60$ Newton. Write down the equation of motion and calculate the period of free oscillations. If a harmonic force $F = 20\cos 20t$ is applied, calculate the amplitude of forced oscillations.

Solution

The Newton's second law

$$ma(t) = -kx(t) - F_{\text{friction}}, \qquad a = \frac{d^2x(t)}{dt^2}.$$

So

$$m\frac{d^2x(t)}{dt^2} + kx(t) = -F_{\text{friction}}$$

$$0.2\frac{d^2x(t)}{dt^2} + 100x(t) = -60$$

$$\frac{d^2x(t)}{dt^2} + 500x(t) = -300.$$

Let $x(t) = \tilde{x}(t) - \frac{3}{5}$, then

$$\frac{d^2\tilde{x}(t)}{dt^2} + 500\tilde{x}(t) = 0.$$

Because equation of motion for oscillator is

$$\frac{d^2\tilde{x}(t)}{dt^2} + \omega^2\tilde{x}(t) = 0.$$

The frequency of free oscillation $\omega = \sqrt{500} = 10\sqrt{5}\ \text{s}^{-1}$.

Thus, the period of free oscillation

$$T = \frac{2\pi}{\omega} = \frac{\pi}{5\sqrt{5}} = \frac{\sqrt{5}\pi}{25}\ \text{s}.$$

In the case when a harmonic force is applied, the equation of motion is

$$\frac{d^2\tilde{x}(t)}{dt^2} + 500\tilde{x}(t) = 100\cos 20t.$$

Let $\tilde{x}(t) = A\cos 20t$, then $-400A\cos 20t + 500A\cos 20t = 100\cos 20t$. So $A = 1$ m.

Answers: $\frac{\sqrt{5}\pi}{25}$ s, 1m.