Answer on #69364, Physics / Molecular Physics | Thermodynamics

Question. It is desired to double the efficiency of a Carnot engine from 35% by raising its temperature of heat addition, while keeping the temperature of heat rejection constant. What percentage of increase in high temperature is required?

Given: $\eta = 35\ \%\ (0.35);$

$T_2 = const;$

$\eta' = 2 \cdot \eta.$

Find: $T_1'.$

Solution. The efficiency of the Carnot engine is defined to be:

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1},$$

where – T_1 – is the absolute temperature of the hot reservoir; T_2 – is the absolute temperature of the cold reservoir. Thus

$$\eta = 1 - \frac{T_2}{T_1};\ \frac{T_2}{T_1} = 1 - \eta;$$

$$T_2 = T_1(1 - \eta),$$

and

$$T_2 = T_1'(1 - \eta').$$

$$T_1(1 - \eta) = T_1'(1 - \eta')$$

$$T_1(1 - \eta) = T_1'(1 - 2\eta);$$

$$\frac{T_1'}{T_1} = \frac{1 - \eta}{1 - 2\eta} = \frac{1 - 0.35}{1 - 2 \cdot 0.35} = \frac{0.65}{0.3} \approx 2.17\ or\ 217\ \%.$$

Answer: $T_1' = 2.17 \cdot T_1$, percentage of increase – $217\ \%$.