Answer on Question #69288-Physics-Other

For a weakly damped harmonic oscillator, the instantaneous displacement is given by

$$x(t) = a_0 exp(-bt)\, cos(\omega_d t + \phi).$$

Calculate its average energy

Solution

$$v(t) = \frac{d}{dt}x(t) = -b\, a_0 exp(-bt) cos(\omega_d t + \phi) - \omega_d a_0 exp(-bt)\, sin(\omega_d t + \phi).$$

$$E = \frac{kx^2}{2} + \frac{mv^2}{2}$$

For a weakly damped harmonic oscillator

$$b \ll \omega_d$$

Velocity with weak damping:

$$v(t) \approx -\omega_d a_0 exp(-bt)\, sin(\omega_d t + \phi).$$

$$E = \frac{k}{2} a_0^2 exp(-2bt) \cos^2(\omega_d t + \phi) + \frac{1}{2} m\omega_d^2 a_0^2 exp(-2bt) \sin^2(\omega_d t + \phi)$$

The average energy is

$$E = \frac{1}{2} k a_0^2 e^{-2bt}$$