A $m = 4.3\ kg$ object traveling at $v = 2.5\ m/s$ to the right collides head-on with a $M = 3.8\ kg$ object which is at rest. If the impact is perfectly elastic, the velocity of the 3.8 kg object after the impact is ____ m/s.

Solution:

The momentum conservation law

$$mv = mv' + Mu$$

The energy conservation law

$$\frac{mv^2}{2} = \frac{mv'^2}{2} + \frac{Mu^2}{2}$$

So

$$v' = v - \frac{M}{m}u$$

$$v^2 = \left(v - \frac{M}{m}u\right)^2 + \frac{M}{m}u^2$$

$$v^2 = v^2 - 2\frac{M}{m}vu + \frac{M^2}{m^2}u^2 + \frac{M}{m}u^2$$

$$u\left(\frac{M^2}{m^2} + \frac{M}{m}\right) = 2\frac{M}{m}v$$

$$u\left(\frac{M}{m} + 1\right) = 2v$$

$$u = \frac{2v}{1 + \frac{M}{m}} = \frac{2\times 2.5}{1 + \frac{3.8}{4.3}} = 2.65\frac{\text{m}}{\text{s}}.$$

Answers: $u = 2.65\frac{\text{m}}{\text{s}}$.