Question 69090 – Physics/Electromagnetism

Derive the wave equation for the z-component of the electric field of an electromagnetic wave.

Answer:

Maxwell's Equations are laws - just like the law of gravity. These equations are rules the universe uses to govern the behaviour of electric and magnetic fields. A flow of electric current will produce a magnetic field. If the current flow varies with time (as in any wave or periodic signal), the magnetic field will also give rise to an electric field. Maxwell's Equations shows that separated charge (positive and negative) gives rise to an electric field - and if this is varying in time as well will give rise to a propagating electric field, further giving rise to a propagating magnetic field.

$$\nabla\times\nabla\times\mathbf{E} = -\nabla^2\mathbf{E}$$

If we start now with Faradays Law, and take the curl of both sides, we get:

$$\nabla\times\nabla\times\mathbf{E} = -\mu\frac{\partial}{\partial t}(\nabla\times\mathbf{H})$$

We can rewrite the left side of equation and we can rewrite the right side of equation by substituting in Amperes Law:

$$\nabla\times\nabla\times\mathbf{E} = -\nabla^2\mathbf{E} = -\mu\frac{\partial}{\partial t}(\nabla\times\mathbf{H}) = -\mu\frac{\partial}{\partial t}\left(\frac{\partial\mathbf{D}}{\partial t}+\mathbf{J}\right) = -\mu\varepsilon\frac{\partial}{\partial t}\left(\frac{\partial\mathbf{E}}{\partial t}\right)$$

The equation is known as the Wave Equation. It is actually 3 equations, since we have an x-, y- and z-component for the **E** field:

$$\nabla^2 E_z = \mu\varepsilon\frac{\partial^2 E_z}{\partial t^2}, \qquad \frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\frac{\partial^2 E_z}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E_z}{\partial t^2}$$