Answer on Question 69059, Physics, Mechanics, Relativity

Question:

1) A disc rolling along a horizontal plane has a moment of inertia 2.5 $kg \cdot m^2$ about its center and a mass of 5 kg. The velocity along the plane is 2 m/s. If the radius of the disc is 1 m:

a) calculate the angular velocity

b) calculate the total energy of the disc

Solution:

a) We can find the angular velocity of the disc from the relationship between the linear and angular variables:

$$v = r\omega,$$

here, v is the linear velocity of the disc, r is the radius of the disk and ω is the angular velocity of the disc.

Then, we get:

$$\omega = \frac{v}{r} = \frac{2\ \frac{m}{s}}{1\ m} = 2\ \frac{rad}{s}.$$

b) The total energy of disc rolling along a horizontal plane consists of the sum of translational kinetic energy and rotational kinetic energy:

$$KE_{disc} = KE_{translational} + KE_{rotational}.$$

The translational kinetic energy can be found from the formula:

$$KE_{translational} = \frac{1}{2}mv^2,$$

here, m is the mass of the disc, v is the linear velocity of the disc.

The rotational kinetic energy can be found from the formula:

$$KE_{rotational} = \frac{1}{2}I\omega^2,$$

here, $I = 2.5\ kg \cdot m^2$ is the moment of inertia of the disc about its center, r is the radius of the disk and ω is the angular velocity of the disc.

Then, we get:

$$KE_{disc} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2, = \frac{1}{2} \cdot \left(5\ kg \cdot \left(2\ \frac{m}{s}\right)^2 + 2.5\ kg \cdot m^2 \cdot \left(2\ \frac{rad}{s}\right)^2\right) = 15\ J.$$

Answer:

a) $\omega = 2\ \frac{rad}{s}$.

b) $KE_{disc} = 15\ J$.

2) A force $\left(6\vec{\imath} + 4\vec{\jmath} - 10\vec{k}\right)\ N$ acts tangentially to the circumference of a disc of radius $\left(2\vec{\imath} + \vec{\jmath} + 3\vec{k}\right)\ m$. Find the torque.

Solution:

By the definition of the torque we get:

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & 1 & 3 \\ 6 & 4 & -10 \end{vmatrix} = \vec{\imath} \cdot \begin{vmatrix} 1 & 3 \\ 4 & -10 \end{vmatrix} - \vec{\jmath} \cdot \begin{vmatrix} 2 & 3 \\ 6 & -10 \end{vmatrix} + \vec{k} \cdot \begin{vmatrix} 2 & 1 \\ 6 & 4 \end{vmatrix} =$$

$$= (-10 - 4 \cdot 3)\vec{\imath} - (2 \cdot (-10) - 6 \cdot 3)\vec{\jmath} + (2 \cdot 4 - 6)\vec{k} =$$

$$= \left(-22\vec{\imath} + 38\vec{\jmath} + 2\vec{k}\right)\ N \cdot m.$$

Also, we can find the magnitude of the torque from the Pythagorean theorem:

$$\tau = \sqrt{\tau_x^2 + \tau_y^2 + \tau_z^2} = \sqrt{(-22\ N \cdot m)^2 + (38\ N \cdot m)^2 + (2\ N \cdot m)^2} = 43.95\ N.$$

Answer:

$\vec{\tau} = \left(-22\vec{\imath} + 38\vec{\jmath} + 2\vec{k}\right)\ N \cdot m$.

The magnitude of the torque is $\tau = 43.95\ N$.