Answer to Question #68457, Physics / Mechanics | Relativity

Question: An ideal monatomic gas undergoes an adiabatic compression from state 1 with pressure p1=1 atm, volume V1=8 L, and temperature T1=300 K to state 2 with pressure p2=32 atm, volume V2=1 L

(a) What is the temperature of the gas in state 2?

(b) How many moles of gas are present?

(c) What is the average translational kinetic energy per mole before and after the compression?

(d) What is the ratio of the squares of the rms speeds before and after the compression?

(e) If we do not know that the ideal gas here is monatomic, demonstrate that the gas is truly monatomic.

Solution:

a) In general for the gas we can write

$$\frac{PV}{T} = const$$

So

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \to \boldsymbol{T_2} = T_1\left(\frac{P_2V_2}{P_1V_1}\right) = 300 * \left(\frac{32}{8}\right) = \mathbf{1200\ K}$$

b) It is known that

$$PV = \nu RT$$

Where ν is a number of moles and R is a gas constant. Considering that 1atm is equal to 10^5 Pa and 1L = 10^{-3}m^3

$$P_1V_1 = \nu RT_1 \to \boldsymbol{\nu} = \frac{P_1V_1}{RT_1} = (10^5 * 8 * 10^{-3})/(8.31 * 300) = \mathbf{0.321\ mole}$$

c) The average translational kinetic energy per mole for a monoatomic gas is calculated as

$$K = \frac{3RT}{2}$$

So

$$\boldsymbol{K_1} = \left(\frac{3}{2}\right)RT_1 = 3 * 300 * \frac{8.31}{2} = \mathbf{3739.5\ J}$$

$$\boldsymbol{K_2} = \left(\frac{3}{2}\right)RT_2 = 3 * 1200 * \frac{8.31}{2} = \mathbf{14958\ J}$$

d) The kinetic energy of the particle in the gas can be calculated as

$$\frac{mv_{rms}^2}{2} = \frac{3}{2}k_bT$$

So

$$\frac{\boldsymbol{v_{rms1}^2}}{\boldsymbol{v_{rms2}^2}} = \frac{T_1}{T_2} = \frac{300}{1200} = \mathbf{\frac{1}{4}}$$

e) If the process is adiabatic then

$$P_1 V_1^\gamma = P_2 V_2^\gamma \rightarrow \left(\frac{V_1}{V_2}\right)^\gamma = \frac{P_2}{P_1}$$

$$8^\gamma = 32$$

$$(2^3)^\gamma = 2^5$$

$$\boldsymbol{\gamma = \frac{5}{3}}$$

A value which **corresponds to an ideal monoatomic gas**