Answer on Question 68208, Physics, Molecular Physics, Thermodynamics

Question:

A block of mass $M = 2.0\ kg$ initially at rest at the edge of a frictionless table height $h = 1.5\ m$ is fired at by a bullet of mass $m = 6\ g$. The bullet embeds in the block, and after impact the block hits the floor a distance $d = 2.3\ m$ from the bottom of the table. Determine the initial speed of the bullet.

Solution:

We can find the initial velocity of the bullet from the law of conservation of momentum:

$$mv_{bullet} + Mv_{block} = (m + M)v_f,$$

here, m is the mass of the bullet, M is the mass of the block, v_{bullet} is the initial velocity of the bullet before the impact with the block, v_{block} is the initial velocity of the block before the impact with the bullet (since the block initially at rest it is equal to zero), v_f is the final velocity of the bullet and block after the impact.

Then, we get:

$$mv_{bullet} = (m + M)v_f \ (1).$$

Let's find the final velocity of the bullet and block after the impact. In order to find it, let's consider the projectile motion of the bullet and block off the table. Let's first find the time that needs the bullet and block to reach the floor from the kinematic equation:

$$h = v_{0y}t + \frac{1}{2}a_y t^2,$$

here, $h = -1.5\ m$ is the displacement of the bullet and block in the vertical y-direction (since the bullet and the block falls down it will be with sign minus), v_{0y} is the vertical component of the initial velocity of the bullet and block (since initially the bullet and block don't move in the vertical direction it is equal to zero), $a_y = g = -9.8\ m/s^2$ is the acceleration due to gravity (since the acceleration due to gravity directed downward it will be with sign minus), t is the time.

Then, we get:

$$h = \frac{1}{2}gt^2,$$

$$t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\cdot(-1.5\ m)}{-9.8\ \frac{m}{s^2}}}=0.553\ s.$$

Then, from the same kinematic equation we can find the initial velocity of the bullet and block in the horizontal direction (x-direction):

$$d=v_{0x}t+\frac{1}{2}a_x t^2,$$

here, $d=2.3\ m$ is the displacement of the bullet and block in the horizontal direction, $v_{0x}=v_0$ is the initial velocity of the bullet and block in the horizontal direction, $a_x=0$ (the acceleration in the horizontal direction will be zero).

So, we get:

$$d=v_0 t,$$

$$v_0=\frac{d}{t}=\frac{2.3\ m}{0.553\ s}=4.16\ \frac{m}{s}.$$

The initial velocity of the bullet and block in the horizontal direction when the block is slid down off the table equals to the final velocity of the bullet and block after the impact:

$$v_f=v_0=4.16\ \frac{m}{s}.$$

Substituting v_f into the equation (1) we can find the initial velocity of the bullet:

$$v_{bullet}=\frac{(m+M)v_f}{m}=\frac{(0.006\ kg+2.0\ kg)}{0.006\ kg}\cdot 4.16\ \frac{m}{s}=1391\ \frac{m}{s}.$$

Answer:

$v_{bullet}=1391\ \frac{m}{s}.$