

Answer on Question #68138 Physics / Other

The sum of two unit vectors is a unit vector, what is the magnitude of difference in two unit vectors?

Solution:

Let us consider two unit vectors \mathbf{a} and \mathbf{b}

$$|\mathbf{a}| = 1, |\mathbf{b}| = 1.$$

Because the sum of these vectors is a unit vector $|\mathbf{a} + \mathbf{b}| = 1$, thus

$$|\mathbf{a} + \mathbf{b}|^2 = (\mathbf{a} + \mathbf{b})^2 = |\mathbf{a}|^2 + 2(\mathbf{a} \cdot \mathbf{b}) + |\mathbf{b}|^2 = |\mathbf{a}|^2 + 2|\mathbf{a}||\mathbf{b}| \cos \varphi + |\mathbf{b}|^2 = 2 + 2 \cos \varphi = 1.$$

Therefore

$$\cos \varphi = -\frac{1}{2}.$$

The magnitude of difference in two unit vectors

$$\begin{aligned} |\mathbf{a} - \mathbf{b}| &= \sqrt{|\mathbf{a} - \mathbf{b}|^2} = \sqrt{(\mathbf{a} - \mathbf{b})^2} = \sqrt{|\mathbf{a}|^2 - 2(\mathbf{a} \cdot \mathbf{b}) + |\mathbf{b}|^2} = \\ &= \sqrt{|\mathbf{a}|^2 - 2|\mathbf{a}||\mathbf{b}| \cos \varphi + |\mathbf{b}|^2} = \sqrt{2 - 2 \cos \varphi} = \sqrt{3}. \end{aligned}$$

Answer: $|\mathbf{a} - \mathbf{b}| = \sqrt{3}$.

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