Answer on Question #68091 Physics / Mechanics | Relativity

The space time equation for S.H.M is $x(t) = x_{\max}\sin(\omega t + \varphi)$. Prove that the velocity v and the acceleration a at any instant are connected by the relation$\omega^2 v^2 + a^2 = \omega^4 x_{\max}^2$.

Solution:

Let us consider the space time equation

$$x(t) = x_{\max}\sin(\omega t + \varphi).$$

The velocity

$$v(t) = x'(t) = \omega x_{\max}\cos(\omega t + \varphi).$$

The acceleration

$$a(t) = v'^{(t)} = x''^{(t)} = -\omega^2 x_{\max}\sin(\omega t + \varphi).$$

So

$$\omega^2 v^2 + a^2 = \omega^2\omega^2 x_{\max}^2\cos^2(\omega t + \varphi) + \omega^4 x_{\max}^2\sin^2(\omega t + \varphi) =$$

$$= \omega^4 x_{\max}^2(\cos^2(\omega t + \varphi) + \sin^2(\omega t + \varphi)) = \omega^4 x_{\max}^2.$$

Answer: $\omega^2 v^2 + a^2 = \omega^4 x_{\max}^2$.