Answer to Question #67612, Physics / Mechanics | Relativity

Question: Define torque. By using its definition, derive a mathematical formula to calculate couple of a rotating system.

Solution: I suppose that in here we are talking about a torque as a free vector, the vector, which is independent of the reference point, unlike for the moment (also called a torque). So then the torque will be defined as

$$\vec{\tau} = \sum_{i}^{N} \vec{F_i} \times \vec{r_i} = const$$

Now lets move the reference point by some $\vec{r}'$ and recalculate the torque

$$\vec{\tau} = \sum_{i}^{N} \vec{F_i} \times (\vec{r_i} + \vec{r}') = \sum_{i}^{N} \vec{F_i} \times \vec{r_i} + \left(\sum_{i}^{N} \vec{F_i} \right) \times \vec{r}'$$

In summary

$$\left(\sum_{i}^{N} \vec{F_i} \right) \times \vec{r}' = 0$$

So

$$\left(\sum_{i}^{N} \vec{F_i} \right) = 0$$

Such a family of forces applied to a body, that

$$\sum_{i}^{N} \vec{F_i} = 0$$

Is called a couple.