Answer on Question 67515, Physics, Molecular Physics, Thermodynamics

Question:

Hydrogen is heated in a vessel to a temperature of $10000\ K$. Let each molecule possess an average energy E_1. A few molecules escape into the atmosphere at $300\ K$. Due to collisions, their energy changes to E_2. Calculate ratio E_1/E_2.

Solution:

The diatomic gas molecule has 7 degrees of freedom (3 translational degrees of freedom, 2 rotational degrees of freedom and 2 vibrational degrees of freedom). At low temperature ($300\ K$) the molecules of the diatomic gas may be thought of as rigid and possessing no vibrational energy (thus, in this case we have 5 degrees of freedom). Applying the law of equipartition of energy (the total kinetic energy of the dynamical system is equally divided among all its degrees of freedom and it is equal to $\frac{1}{2}kT$ for each degree of freedom), we can find the average energy of each molecule of diatomic gas at temperature $T_2 = 300\ K$:

$$E_2 = 3 \cdot \left(\frac{1}{2}kT_2\right) + 2 \cdot \left(\frac{1}{2}kT_2\right) = \frac{3}{2}kT_2 + kT_2 = \frac{5}{2}kT_2.$$

here, k is the Boltzmann constant.

At very high temperatures ($10000\ K$), vibrational motion of the molecules can't be neglected (thus, in this case we have 7 degrees of freedom). So, again applying the law of equipartition of energy, we can find the average energy of each molecule of diatomic gas at temperature $T_1 = 10000\ K$:

$$E_1 = 3 \cdot \left(\frac{1}{2}kT_1\right) + 2 \cdot \left(\frac{1}{2}kT_1\right) + 2 \cdot \left(\frac{1}{2}kT_1\right) = \frac{3}{2}kT_1 + 2kT_1 = \frac{7}{2}kT_1.$$

Finally, we can find the ratio E_1/E_2:

$$\frac{E_1}{E_2} = \frac{\frac{7}{2}kT_1}{\frac{5}{2}kT_2} = \frac{7}{2} \cdot \frac{2}{5} \cdot \frac{T_1}{T_2} = \frac{7}{5} \cdot \frac{10000\ K}{300\ K} = \frac{140}{3} = 46.6.$$

Answer:

$$\frac{E_1}{E_2} = 46.6.$$

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