Answer on Question #66810, Physics / Molecular Physics | Thermodynamics

Using the first law of the thermodynamics for an adiabatic processes, establish the relation PV power of gama =K Where gama is the ratio pf the heat capacity of constant pressure to that at constant volume .plot this equation on a P-v diagram .What will be its slope?

Solution:

According to the first law of thermodynamics,

$\mathrm{dU} + \delta\mathrm{W} = \delta\mathrm{Q} = 0$ (1),

where *dU* is the change in the internal energy of the system and *δW* is work done by the system. Any work (*δW*) done must be done at the expense of internal energy *U*, since no heat *δQ* is being supplied from the surroundings.

Pressure–volume work *δW* done by the system is defined as

$\delta\mathrm{W} = \mathrm{PdV}$ (2),

However, *P* does not remain constant during an adiabatic process but instead changes along with *V*. It is desired to know how the values of *dP* and *dV* relate to each other as the adiabatic process proceeds. For an ideal gas the internal energy is given by

$\mathrm{U} = \alpha\mathrm{nRT}$ (3),

where *α* is the number of degrees of freedom divided by two, *R* is the universal gas constant and *n* is the number of moles in the system (a constant).

Differentiating Equation (3) and use of the ideal gas law, *PV* = *nRT*, yields

$\mathrm{U} = \alpha\mathrm{nRT} = \alpha\mathrm{d(PV)} = \alpha(\mathrm{PdV} + \mathrm{VdP})$ (4)

Equation (4) is often expressed as $dU = nC_V\, dT$ because $C_V = \alpha R$.

Now substitute equations (2) and (4) into equation (1) to obtain

$-\mathrm{PdV} = \alpha\mathrm{PdV} + \alpha\mathrm{VdP}(\mathrm{PdV} + \mathrm{VdP})$ (5)

factorize -*P dV*:

$-(\alpha + 1)\mathrm{PdV} = \alpha\mathrm{VdP}$ (6)

and divide both sides by *PV*:

$-(\alpha + 1)\frac{\mathrm{dV}}{\mathrm{V}} = \alpha\frac{\mathrm{dP}}{\mathrm{P}}$ (7)

After integrating the left and right sides from V_0 to *V* and from P_0 to *P* and changing the sides respectively,

$\ln\frac{\mathrm{P}}{\mathrm{P_0}} = \frac{\alpha+1}{\alpha}\ln\frac{\mathrm{V}}{\mathrm{V_0}}$ (8)

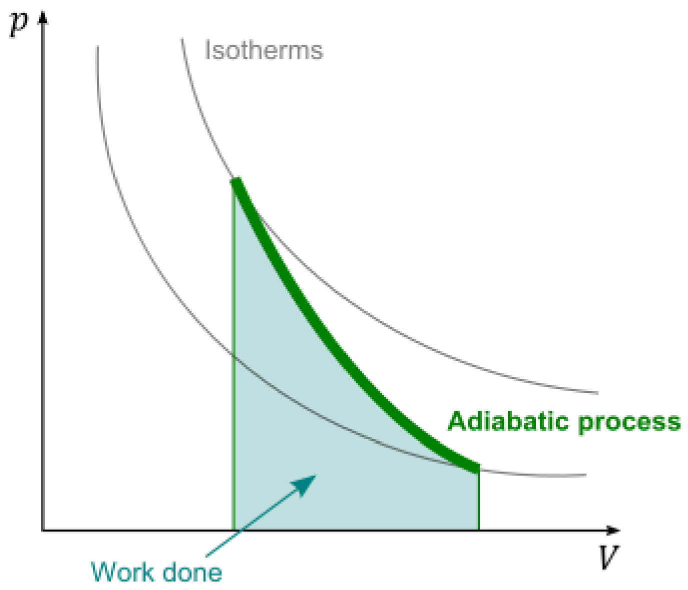
Of (8) $\Rightarrow \frac{\mathrm{P}}{\mathrm{P_0}} = \left(\frac{\mathrm{V}}{\mathrm{V_0}}\right)^{-\gamma}$ (9),

where $\gamma = \frac{\alpha+1}{\alpha}$

Therefore,

$$\left(\frac{P}{P_0}\right) \times \left(\frac{V}{V_0}\right)^{\gamma} = 1 \ (10)$$

Of (10) $\Rightarrow \ P_0 V_0^{\gamma} = PV^{\gamma} = \text{const}$



Adiabat on PV diagram is a "steep hyperbola",

the slope is a tangent of angle,

tangent of angle for adiabat is greater than tangent of angle for isotherms (adiabat is steeper isotherms)

Answer provided by https://www.AssignmentExpert.com