Answer on Question #66747-Physics-Molecular Physics-Thermodynamics

Define mean free path of a molecules in a gas. Derive the law of distribution of free path.

Solution

The mean free path is the average distance traveled by a moving molecule between successive collisions, which modify its direction or energy or other particle properties.

Consider a large number of molecules at a certain instant. As they travel they will collide among themselves and with other molecules. We wish to estimate the number that has not made a collision at some later time. Let the number of molecules surviving a collision in travelling distance r be N. If each molecule is allowed to travel a further distance dx, more collisions will occur. We assume that the number of collisions is proportional to the number of molecules N, and the distance dx. That is, the number of molecules removed by these collisions will be proportional to Ndx. Since the number of molecules decreases with increasing distance, we can write

$$dN = -P_c N dx$$

where P_c is a constant of proportionality and is called the Collision probability. One can rewrite the above equation as

$$\frac{dN}{N} = -P_c dx$$

This can be integrated to

$$N = N_0 e^{-P_c x}$$

where N_0 is the number of molecules at $x = 0$.

From this equation we find that number of molecules surviving a collision decreases exponentially. Further, the probability that a gas molecule will cover a distance x without making any collision is

$$\frac{N}{N_0} = f(x) = e^{-P_c x}.$$

This is the law of distribution of free paths.

$P_c = \frac{1}{l}$, where l is mean free path.

Thus

$$\frac{N}{N_0} = e^{-\frac{x}{l}}.$$