Answer on Question #66743, Physics / Mechanics | Relativity

Question:

A satellite going around the earth in an elliptical orbit has a speed of 10 km/s at the perigee which is at a distance of 227 km from the surface of the earth. Calculate the apogee distance & its speed at that point.

Solution:

The satellite's speed at the perigee may be calculated according to this formula:

$v_p = \sqrt{\frac{GM}{r_p}(1+e)}$, where G — gravitational constant (6.674×10^{-11} $m^3kg^{-1}s^{-2}$);

M — the Earth's mass (5.972×10^{24} kg);

r_p — perigee distance from the centre of the earth;

e — excentricity of the orbit.

Then $e = \frac{v_p^2 r_p}{GM} - 1 = \frac{10000^2\cdot(6371000+227000)}{6.674\times10^{-11}\cdot 5.972\times10^{24}} - 1 = 0.655$

In apogee the satellite's speed:

$v_a = \sqrt{\frac{GM}{r_a}(1-e)}$, where r_a — apogee distance from the centre of the earth.

For elliptical orbit the distances r_p and r_a are related as $\frac{r_a}{r_p} = \frac{1+e}{1-e}$, and then

$r_a = r_p\frac{1+e}{1-e} = (6371000+227000)\frac{1+0.655}{1-0.655} = 31651275\ m$.

The apogee distance from the surface of the earth

$d_a = 31651275 - 6371000 = 25280275\ m \cong 25280\ km.$

$v_a = \sqrt{\frac{6.674\times10^{-11}\ \cdot\ 5.972\times10^{24}}{31651275}(1-0.655)} = 2084\frac{m}{s} \cong 2.1\ km/s$

Answer:

$25280\ km$

$2.1\ km/s$