For a damped harmonic oscillator, the equation of motion is $(/)(/)0\;2\;2m\;d\;x\;dt + g\;dx\;dt + kx =$ with m = 0.50 kg, g = 0.70 kgs−1 and k = 70 Nm−1. Calculate (i) the period of motion,(ii) number of oscillations in which its amplitude will become half of its initial value,(iii) the number of oscillations in which its mechanical energy will drop to half of its initial value, (iv) its relaxation time, and (v) quality factor

Solution:

$$m\frac{d^2x}{dt^2} + g\frac{dx}{dt} + kx = 0 \;(1),$$

where m=0.5 kg, g=0.7 kg×s^{-1}, k=70 N×m^{-1}

Of (1) $\Rightarrow \frac{d^2x}{dt^2} + \frac{g}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$ (2)

Of (2) $\Rightarrow$ cyclic frequency of free oscillations: $\omega_0^2 = \frac{k}{m}$ (3)

Of (2) $\Rightarrow$ damped coefficient: $\beta = \frac{g}{2m}$ (4)

Solution of equation (2):

$x(t) = Ae^{-\beta t}\cos\omega t$ (5),

where cyclic frequency of free damped oscillations $\omega = \sqrt{\omega_0^2 - \beta}$

(i) the period of motion:

$T = \frac{2\pi}{\omega_0}$ (6)

Of (3) $\Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$ (7)

(7) in (6): $T = 2\pi\sqrt{\frac{m}{k}}$ (8)

Of (8) $\Rightarrow$ T=0.53 s (9)

(ii)

$\frac{x(0)}{x(t)} = 2$ (10)

Of (10) $\Rightarrow e^{\beta t} = 2$ (11)

Of (11) $\Rightarrow t = \frac{1}{\beta}\ln 2$ (12)

(4) in (12): $t = \frac{2m}{g}\ln 2$ (13)

Of (13) $\Rightarrow$ t=0.99 s (14)

Number of oscillations in which its amplitude will become half of its initial value:

$N = \frac{t}{T}$ (15)

(9) and (14) in (15): N=1.87

(iii)

$$\frac{x(0)}{x(t)} = \sqrt{2} \text{ (16)}$$

Of (16) $\Rightarrow$ $e^{\beta t} = 1.41$ (17)

Of (17) $\Rightarrow$ $t = \frac{1}{\beta} \ln 1.41$ (18)

(4) in (18): $t = \frac{2m}{g} \ln 1.41$ (19)

Of (19) $\Rightarrow$ t=0.49 s (20)

Number of oscillations in which its mechanical energy will drop to half of its initial value:

$$N = \frac{t}{T} \text{ (21)}$$

(9) and (20) in (21): N=0.93

(iv)

its relaxation time:

$$\tau = \frac{1}{\beta} \text{ (22)}$$

(4) in (22): $\tau = \frac{2m}{q}$ (23)

Of (23) $\Rightarrow$ $\tau = 1.43$ s

(v)

quality factor:

$$Q = \frac{\pi}{\beta T} \text{ (23)}$$

(4) in (23): $Q = \frac{2\pi m}{gT}$ (24)

(9) in (24): Q=8.46

Answer:

        (i) T=0.53 s

        (ii) N=1.87

        (iii) N=0.93

        (iv) $\tau = 1.43$ s

        (v) Q=8.46