Answer on Question #66346, Physics / Mechanics | Relativity

Use the Frobenius method to obtain one solution of the following ODE: 4 yx '' + 2y' + y = 0

Solution:

$$a_{k+1} = -a_k / (2k + 2r + 2)(2k + 2r + 1), k = 0, 1, 2... (1)$$

To find y_1 apply (14) with $r = r_1 = 1 / 2$ to get the recurrence relation

$$a_{k+1} = -a_k /(2k + 3)(2k + 2), k = 0, 1, 2...$$

Then

$$a_1 = -a_0 /3 \cdot 2 , a_2 = -a_1 / 5 \cdot 4 , a_3 = -a_2 / 7 \cdot 6$$

$$\text{So } a_1 = - a_0 / 3! , a_2 = a_0 / 5! , a_3 = - a_0 / 7!$$

Since a_0 is arbitrary, let $a_0 = 1$

$$a_k(r_1) = (-1)^k / (2k + 1)! , k = 0, 1, 2...$$

$$y_1(x) = x^{1/2}\sum_{k=0}^{\infty} ((-1)^k / (2k + 1)!) x^k$$

To find y_2, just apply (1) with $r = r_2 = 0$ to get the recurrence relation

$$b_{k+1} = -b^k / (2k + 2)(2k + 1)$$

Letting the arbitrary constant $b_0 = 1$, then

$$b_k(r_2) = (-1)^k / (2k)$$

$$y_2(x) = \sum_{k=0}^{\infty} ((-1)^k /(2k)) x^k$$