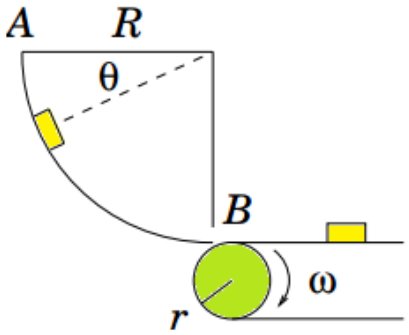
Answer on Question #66182, Physics / Mechanics | Relativity

Particles are released from rest at A and slide down the smooth surface of height h to a conveyor B. The correct angular velocity (0 of the conveyor pulley of radius r to prevent any sliding on the belt as the particles transfer to the conveyor is

Solution:



We write the equations of motion using polar coordinates

$$mg\ cos\theta = mR\dot{\theta}$$

$$-N + mg\sin\theta = -mR\dot{\theta}^2$$

Integrate the first equation

$$\int_0^{\dot{\theta}} \dot{\theta} d\dot{\theta} = \frac{g}{R}\int_0^{\theta} cos\theta d\theta$$

$$\frac{\dot{\theta}^2}{2} = \frac{g}{R}\sin\theta$$

$$\dot{\theta} = \sqrt{\frac{2g}{R}\sin\theta}$$

The conveyor pulley must turn at the rate

$$R\dot{\theta} = r\omega$$

For θ = π/2, so that

$$\omega = \sqrt{\frac{2gR}{r}}$$

Answer: $\boldsymbol{\omega = \sqrt{\frac{2gR}{r}}}$