Answer on Question #66036, Physics / Mechanics | Relativity

The position vector of two particles of mass 4.0 kg and 2.0 kg are, respectively, $\vec{\mathbf{r}}_1 = 3t\hat{\mathbf{i}} + t\hat{\mathbf{j}} + 2t^2\hat{\mathbf{k}}$ and $\vec{\mathbf{r}}_2 = 3\hat{\mathbf{i}} + (t^2-1)\hat{\mathbf{j}} + 4t\hat{\mathbf{k}}$ where t is in seconds and the position in metres. Determine the position vector of the centre of mass of the system, the velocity of the cm and the net force acting on the system.

Solution:

The center of mass is located at

$$\vec{\mathbf{r}}_{CM} = \left(\frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2}\right)$$

$$\vec{\mathbf{r}}_{CM} = \left(\frac{4\left(3t\hat{\mathbf{i}} + t\hat{\mathbf{j}} + 2t^2\hat{\mathbf{k}}\right) + 2\left(3\hat{\mathbf{i}} + (t^2-1)\hat{\mathbf{j}} + 4t\hat{\mathbf{k}}\right)}{4+2}\right) =$$

$$= \frac{12t\hat{\mathbf{i}} + 3t\hat{\mathbf{j}} + 8t^2\hat{\mathbf{k}} + 6\hat{\mathbf{i}} + 2(t^2-1)\hat{\mathbf{j}} + 8t\hat{\mathbf{k}}}{6} =$$

$$= (2t\hat{\mathbf{i}} + \hat{\mathbf{i}}) + \frac{1}{6}(3t\hat{\mathbf{j}} + 2(t^2-1)\hat{\mathbf{j}}) + \frac{1}{6}\left(8t^2\hat{\mathbf{k}} + 8t\hat{\mathbf{k}}\right) =$$

$$= (2t+1)\hat{\mathbf{i}} + \frac{1}{6}(2t^2 + 3t - 2)\hat{\mathbf{j}} + \frac{4}{3}t(t+1)\mathbf{k}$$

The velocity of the cm is

$$\vec{v}_{cm} = \frac{d\vec{\mathbf{r}}_{CM}}{dt} = 2\hat{\mathbf{i}} + \frac{1}{6}(4t+3)\hat{\mathbf{j}} + \frac{4}{3}(2t+1)\hat{\mathbf{k}}$$

The net force acting on the system

$$\vec{F} = \frac{d\vec{\mathbf{v}}_{CM}}{dt} = \frac{4}{6}\hat{\mathbf{j}} + \frac{8}{3}\hat{\mathbf{k}} = \frac{2}{3}\hat{\mathbf{j}} + \frac{8}{3}\hat{\mathbf{k}}$$