

Answer on Question #65640, Physics / Mechanics | Relativity

The linear density of a vibrating string is $1.3 \times 10^{-4} \text{ kg m}^{-1}$. A transverse wave is propagating on the string and is described by the equation $y(x, t) = 0.021 \sin(30t - x)$ where x and y are in metres and t is in seconds. Calculate the tension in the string.

Find: $T - ?$

Given:

$$\rho = 1.3 \times 10^{-4} \text{ kg} \times \text{m}^{-3}$$

$$y(x, t) = 0.021 \sin(30t - x)$$

Solution:

Wave velocity:

$$v = \sqrt{\frac{T}{\rho}} \quad (1), \text{ where } T \text{ is tension, } \rho \text{ is density}$$

$$\text{Of (1)} \Rightarrow v^2 = \frac{T}{\rho} \quad (2)$$

$$\text{Of (2)} \Rightarrow T = v^2 \times \rho \quad (3)$$

Wave velocity:

$$v = f \times \lambda \quad (4)$$

Linear frequency:

$$f = \frac{\omega}{2\pi} \quad (5), \text{ where } \omega \text{ is cyclic frequency}$$

Wavelength:

$$\lambda = \frac{2\pi}{k} \quad (6), \text{ where } k \text{ is wave number}$$

(5) and (6) in (4):

$$v = \frac{\omega}{2\pi} \times \frac{2\pi}{k} = \frac{\omega}{k} \quad (7)$$

Equation of plane wave:

$$y(x, t) = A \sin(\omega t - x) \quad (8)$$

From the condition of the task:

$$y(x, t) = 0.021 \sin(30t - x) \quad (9)$$

$$\text{Of (8) and (9)} \Rightarrow \omega = 30 \text{ s}^{-1}, k = 1 \text{ m}^{-1} \quad (10)$$

$$(10) \text{ in (7): } v = 30 \text{ m} \times \text{s}^{-1} \quad (11)$$

$$(11) \text{ in (3)} T = 0.117 \text{ N}$$

Answer:

0.117 N