Answer on Question #65635, Physics / Mechanics | Relativity |

For a damped harmonic oscillator, the equation of motion is

$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=0$$

with m = 0.50 kg, γ = 0.70 kg/s and k = 70 N/m.

Calculate (i) the period of motion, (ii) number of oscillations in which its amplitude will become half of its initial value, (iii) the number of oscillations in which its mechanical energy will drop to half of its initial value, (iv) its relaxation time, and (v) quality factor.

Solution

Lets rewrite the equation of motion on this form:

$$\frac{d^2x}{dt^2}+\frac{\gamma}{m}\frac{dx}{dt}+\frac{k}{m}x=0, \qquad \omega_0^2=\frac{k}{m}, \quad \beta=\frac{\gamma}{2m}.$$

Its solution:

$$x(t)=Ae^{-\beta t}\cos\omega t\,, \quad \omega=\sqrt{\omega_0^2-\beta^2}.$$

(i) The period of motion is (in the case of $\beta\to 0$!):

$$T=\frac{2\pi}{\omega_0}=2\pi\sqrt{\frac{m}{k}}.$$

(ii)

$$\frac{x(0)}{x(t)}=2 \;\Rightarrow\; e^{\beta t}=2 \;\Rightarrow\; t=\frac{1}{\beta}ln2\approx 0.69\frac{2m}{\gamma}.$$

$$N=\frac{t}{T}=\frac{0.69\cdot 2m\sqrt{k}}{\gamma\cdot 2\pi\sqrt{m}}=\frac{0.69\sqrt{mk}}{\pi\gamma}.$$

(iii)

$$\frac{x(0)}{x(t)}=\sqrt{2}\approx 1.41 \;\Rightarrow\; e^{\beta t}=1.41 \;\Rightarrow\; t=\frac{1}{\beta}ln1.41\approx 0.34\frac{2m}{\gamma}.$$

$$N=\frac{t}{T}=\frac{0.34\cdot 2m\sqrt{k}}{\gamma\cdot 2\pi\sqrt{m}}=\frac{0.34\sqrt{mk}}{\pi\gamma}.$$

(iv)

$$\tau=\frac{1}{\beta}=\frac{2m}{\gamma}.$$

(v)

$$Q=\frac{\pi}{\beta T}=\frac{m2\pi\sqrt{k}}{\gamma 2\pi\sqrt{m}}=\frac{\sqrt{mk}}{\gamma}.$$

Answers: (i) $2\pi\sqrt{\frac{m}{k}}$, (ii) $\frac{0.69\sqrt{mk}}{\pi\gamma}$, (iii) $\frac{0.34\sqrt{mk}}{\pi\gamma}$, (iv) $\frac{2m}{\gamma}$, (v) $\frac{\sqrt{mk}}{\gamma}$.