Answer on Question #65602 – Physics - Mechanics - Relativity

Condition:

A proton undergoes a head on elastic collision with a particle of unknown mass which is initially at rest and rebounds with 16/25 of its initial kinetic energy. Calculate the ratio of the unknown mass with respect to the mass of the proton.

Solution:

The law of conservation of energy:

$$E_p = E_p' + E_x$$

where E_p is energy of proton before the collision: $E_p = \frac{m_p V_p^2}{2}$,

E_p' - is energy of proton after the collision: $E_p^{'} = \frac{m_p V_p^{'2}}{2}$,

E_x - is energy of particle after the collision

$$E_p' = \frac{16}{25}E_p \rightarrow \frac{m_p V_p'^2}{2} = \frac{16}{25}\frac{m_p V_p^2}{2} \rightarrow V_p'^2 = \frac{16}{25}V_p^2 \rightarrow V_p' = \frac{4}{5}V_p$$

$$E_x = E_p - E_p' = E_p - \frac{16}{25}E_p = \frac{9}{25}E_p \rightarrow \frac{m_x V_x^2}{2} = \frac{9}{25}\frac{m_p V_p^2}{2} \rightarrow m_x V_x^2 = \frac{9}{25}m_p V_p^2$$

The conservation of the total momentum demands that the total momentum before the collision is the same as the total momentum after the collision:

$$m_p V_p = m_p V_p' + m_x V_x \rightarrow m_x V_x = m_p(V_p - V_p') = m_p\left(V_p - \frac{4}{5}V_p\right) = \frac{1}{5}m_p V_p$$

$$\begin{cases} (1) m_x V_x^2 = \frac{9}{25}m_p V_p^2 \\ (2) m_x V_x = \frac{1}{5}m_p V_p \end{cases} \rightarrow \frac{(1)}{(2)} \coloneqq V_x = \frac{9}{5}V_p$$

From (2): $\frac{m_x}{m_p} = \frac{V_p}{5V_x} = \frac{V_p}{5*\frac{9}{5}*V_p} = \frac{1}{9}$

Answer: $\frac{m_x}{m_p} = \frac{1}{9}$